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A 3d approach to numerical dosimetry in quasi-static conditions: problems and example of solutions

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Summary

- Approach to resolution of quasi-static dosimetric problems in 3D.
- Standard approach.
- Overview on main features and critical aspects.
- Details not explained in litterature.

Quasi-static dosimetric analysis: system layout

Body model

Numerical methods applied in the segmented body model region

Post processing

Field distribution in the body model volume and/or boundary conditions on body surface

•Quasi-static fields can be computed separately from the problem's solution **inside** the body model volume.

Source/s model

Results

System layout: exposure to magnetic field only

σ

SPFD

$$\Phi_{n0} = \left(\frac{1}{\sum_{i=1}^{6} \bar{\sigma}_{mi}} \right) \cdot \sum_{i=1}^{6} \left\{ \bar{\sigma}_{mi} \left[\Phi_{ni} - (-1)^i j\omega l_c A_{mi} \right] \right\}$$

Post processing

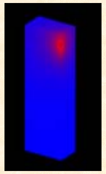
$$J_{x0} = -\sigma_0 \cdot \left(\frac{\Phi_1 - \Phi_0}{l_c} + j\omega A_{m1} \right)$$

$$J_{y0} = -\sigma_0 \cdot \left(\frac{\Phi_3 - \Phi_0}{l_c} + j\omega A_{m3} \right)$$

$$J_{z0} = -\sigma_0 \cdot \left(\frac{\Phi_5 - \Phi_0}{l} + j\omega A_{m5} \right)$$

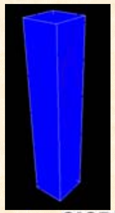
Contents of presentation:

1. Basics of the SPFD method
2. Discretization
3. Example of implementation: overview and critical aspects



$\vec{A}$

Current paths



$|\vec{J}|$

SPFD: basic assumptions

Quasi static conditions:

1. $\lambda >> L$ where L is the problem max. trasversal dimension, usually 2m are taken for L (always true if are true the next conditions)
2. The internal magnetic field has not to be influenced by induced currents
3. For every tissue (or at leas for the "main" ones) must be valid that $\sigma >> \omega\varepsilon$ **(good conductors)**

SPFD

The SPFD equation can be written down in a general form that goes beyond the quasi-static assumption:

$$\nabla \times \vec{E}_i = -j\omega\, \vec{B}_i = -j\omega\, \nabla \times \vec{A}_i$$

$$\nabla \times \left(\vec{E}_i + j\omega\, \vec{A}_i\right) = 0$$

$$\vec{E}_i + j\omega\, \vec{A}_i = -\nabla\Phi$$

$$\vec{E}_i = -\nabla\Phi - j\omega\, \vec{A}_i$$

SPFD

$$\nabla \times \vec{H}_i = (\sigma + j\omega\varepsilon)\vec{E}_i$$

$$\nabla \cdot \left(\nabla \times \vec{H}_i\right) = 0 = \nabla \cdot \left((\sigma + j\omega\varepsilon)\vec{E}_i\right)$$

$$\vec{E}_i = -\nabla\Phi - j\omega\, \vec{A}_i$$

SPFD equation in the general form

$$\nabla \cdot \left[(\sigma + j\omega\varepsilon)\left(-\nabla\phi - j\omega\vec{A}\right)\right] = 0$$

SPFD in quasi static conditions

$$\nabla \cdot \left[(\sigma + j\omega\varepsilon)\left(-\nabla\phi - j\omega\vec{A}\right)\right] = 0$$

$\sigma >> \omega\varepsilon$

$\vec{B}_{\text{internal}} \approx \vec{B}_{\text{sources}}$

$\vec{A}_{\text{internal}} \approx \vec{A}_{\text{sources}}$

The permettivity distribuction plays no further role in the present analysis

$$\nabla \cdot \left(\sigma\left(-\nabla\phi - j\omega\vec{A}_{sources}\right)\right) = 0$$

SPFD: boundary conditions on body surface

- If an external electric field is considered, on body surface must be imposed the condition:

$$\sigma \vec{E}_i \cdot \hat{n} = j\omega\rho_s \qquad \text{(Charge continuity)}$$

Where ρ_s is a surface charge density that can be calculated solving a static potential problem (Laplace problem) with the body surface replaced by a perfect conductor (a **surface** charge density can be assumed only if $\sigma >> \omega\varepsilon$).

- If external magnetic field only is considered, no supplementary boundary conditions are needed on body surface. For good conductors the charge continuity conditions become the current continuity espression that is equivalent to the SPFD expression itself.

$$\nabla \cdot \left(\sigma \left(-\nabla\phi - j\omega\vec{A}_{sources}\right)\right) = 0 \qquad \nabla \cdot \left(\sigma \vec{E}_i\right) = \nabla \cdot \left(\vec{J}_i\right) = 0$$

SPFD in quasi static conditions: current density expression

Once scalar potential has been calculated:

$$\vec{J} = \sigma \vec{E} = \sigma \left(-\nabla\phi - j\omega\vec{A}_{sources}\right)$$

Discretization of the SPFD scalar equation

SPFD integral and differential form in quasi-static conditions

The discretized form of the scalar relation given by the SPFD method can be obtained starting from both the differential and the integral form.

SPFD differential equation :

$$\nabla \cdot \left(\sigma \left(-\nabla\phi - j\omega\vec{A}_{sources}\right)\right) = 0$$

SPFD integral equation :

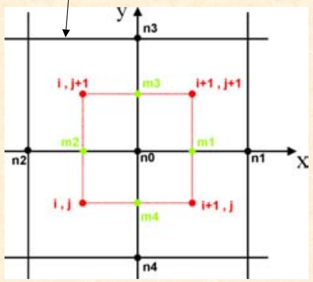
$$\oiint_S \left[(\sigma)\left(-\nabla\phi - j\omega\vec{A}_{sources}\right)\right] d\vec{S} = 0$$

Closed surface

SPFD: 2D discretization

- Unknown scalar potential are applied on cell's edges
- We want to write down the expression of the scalar potential Φ_0 in n_0 in terms of:
  1. Unknown potential Φ_i on neighbouring edges n_i (i=1..4).
  2. Vector potential (magnetic field sources).
  3. Cell's conductivities.

Every black edged square represents a cell with its own conductivity



SPFD: 2D discretization

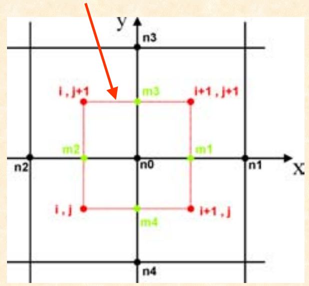
$$\oiint_S \left[(\sigma)\left(-\nabla\phi - j\omega\vec{A}_{sources}\right)\right] d\vec{S} = 0$$

The discretized relation we are searching for can be obtained from the integral form. The flux of the quantity

$$(\sigma)\left(-\nabla\phi - j\omega\vec{A}_{sources}\right)$$

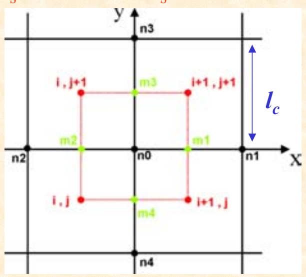
Can be expressed as the sum of the streams though the four edges of the red contour. Since every red edge overlap two cells the mean conductivity is considered. The sources generated vector potential is applied in points m_i (i=1..4). Only the component perpendicular to the red edge is considered.

Closed contour



SPFD: 2D discretization

$$\oiint_S \sigma\nabla\phi\, d\vec{S} = -j\omega \oiint_S \sigma\, \vec{A}_{sources}\, d\vec{S} \qquad l_c \sum_{i=1}^{4}\left(\sigma_{mi}\nabla\phi_{mi}\cdot\hat{n}_{mi}\right) = -j\omega\, l_c \sum_{i=1}^{4}\left(\sigma_{mi}\vec{A}_{mi}\cdot\hat{n}_{mi}\right)$$



Considering:

$$\bar{\sigma}_{m1} = \frac{\sigma_{i+1,j+1}+\sigma_{i+1,j}}{2} \qquad \bar{\sigma}_{m3} = \frac{\sigma_{i+1,j+1}+\sigma_{i,j+1}}{2}$$

$$\bar{\sigma}_{m2} = \frac{\sigma_{i,j+1}+\sigma_{i,j}}{2} \qquad \bar{\sigma}_{m4} = \frac{\sigma_{i,j}+\sigma_{i+1,j}}{2}$$

and:

$$A_{m1} = \vec{A}(m_1)\cdot\hat{i}_x \qquad A_{m3} = \vec{A}(m_3)\cdot\hat{i}_y$$

$$A_{m2} = \vec{A}(m_2)\cdot\hat{i}_x \qquad A_{m4} = \vec{A}(m_4)\cdot\hat{i}_y$$

where:

$$\hat{n}_{m1} = \hat{i}_x \qquad \hat{n}_{m3} = \hat{i}_y$$

$$\hat{n}_{m2} = -\hat{i}_x \qquad \hat{n}_{m4} = -\hat{i}_y$$

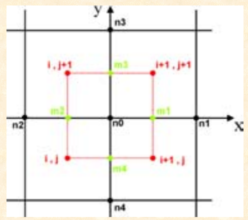
$$\bar{\sigma}_{m1}\frac{\Phi_{n1}-\Phi_{n0}}{lc} + \bar{\sigma}_{m2}\frac{\Phi_{n2}-\Phi_{n0}}{lc} + \bar{\sigma}_{m3}\frac{\Phi_{n3}-\Phi_{n0}}{lc} + \bar{\sigma}_{m4}\frac{\Phi_{n4}-\Phi_{n0}}{lc} =$$

$$= -j\omega\left(\bar{\sigma}_{m1}A_{m1} - \bar{\sigma}_{m2}A_{m2} + \bar{\sigma}_{m3}A_{m3} - \bar{\sigma}_{m4}A_{m4}\right)$$

SPFD: 2D discretization

The last expression is the SPFD discretized form, that can be rewritten in the following ways:

$$\Phi_{n0} = \left(\frac{1}{\sum_{i=1}^{4}\bar{\sigma}_{mi}}\right)\cdot\sum_{i=1}^{4}\left\{\bar{\sigma}_{mi}\left[\Phi_{ni} - (-1)^i j\omega\, l_c A_{mi}\right]\right\}$$



$$\bar{\sigma}_{m1}\cdot\Phi_{n1} + \bar{\sigma}_{m2}\cdot\Phi_{n2} + \bar{\sigma}_{m3}\cdot\Phi_{n3} + \bar{\sigma}_{m4}\cdot\Phi_{n4} - \left(\sum_{i=1}^{4}\bar{\sigma}_{mi}\right)\cdot\Phi_{n0} =$$

$$= -j\omega\cdot l_c\left(\bar{\sigma}_{m1}A_{m1} - \bar{\sigma}_{m2}A_{m2} + \bar{\sigma}_{m3}A_{m3} - \bar{\sigma}_{m4}A_{m4}\right)$$

Usually $\Phi^* = j\Phi$ so that the new discretized equation is scalar.

SPFD: 2D discretization

The last expressions can be obtained also applying the finite difference expressions to the expanded differential form, that is:

$$\nabla \cdot \left(\sigma \left(-\nabla\phi - j\omega \vec{A}_{sources}\right)\right) = 0$$

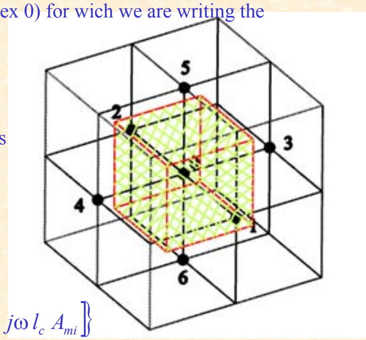
$$\downarrow$$

$$\frac{\partial\sigma}{\partial x}\frac{\partial\Phi_x}{\partial x} + \frac{\partial\sigma}{\partial y}\frac{\partial\Phi_y}{\partial y} + \sigma\left(\frac{\partial^2\Phi_x}{\partial x^2} + \frac{\partial^2\Phi_y}{\partial y^2}\right) = -j\omega\left(\frac{\partial\sigma}{\partial x}A_x + \sigma\frac{\partial A_x}{\partial x} + \frac{\partial\sigma}{\partial y}A_y + \sigma\frac{\partial A_y}{\partial y}\right)$$

That aproach leads to the same discretized expressions but entails more algebric operation.

SPFD: 3D discretization

- The "shifted" voxel, that is the closed surface of the integral SPFD, espression is centered in the voxel's vertex (index 0) for wich we are writing the discretized SPFD expression.
- The 3D discretization is a simple extension of the 2D case
- The means of the conuctivities are done on 4 cells instead than two cells as in the 2D case.
- For every "central" vertex (index 0), six neighbouring vertex have to be considered



$$\Phi_0 = \left(\frac{1}{\sum_{i=1}^{6}\bar{\sigma}_{mi}}\right) \cdot \sum_{i=1}^{6}\left\{\bar{\sigma}_{mi}\left[\Phi_i - (-1)^i\, j\omega\, l_c\, A_{mi}\right]\right\}$$

Post-processing

$$\vec{J}_{i,j} = \sigma_{i,j} \cdot \left(E_{m1} \cdot \hat{i}_x + E_{m3} \cdot \hat{i}_y\right)$$

where:

$$E_{m1} = -\left(\frac{\Phi^*_{i+1,j} - \Phi^*_{i,j}}{l_c} + \omega\, A_{m1}\right)$$

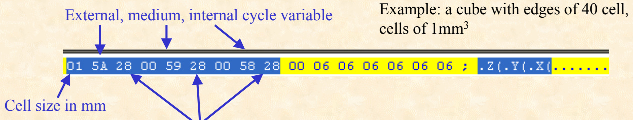
$$E_{m3} = -\left(\frac{\Phi^*_{i,j+1} - \Phi^*_{i,j}}{l_c} + \omega\, A_{m3}\right)$$

Different post-processing approaches are appliable.

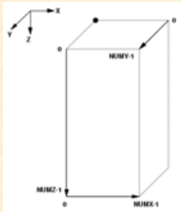
Example of implementation: overview and critical aspects

Body models: dimensions, files organization and memory occupation

Four model are available with 58 recognized tissues. One byte is required for every cell of the model. Cell's position is distilled from the byte position. The structure of the file is described in file's heading.



Model	NUMX	NUMY	NUMZ	n_cells	Model memory occupation [Mb]
Head 1mm	178	235	211	8826130	8,42
Man 3mm	196	114	626	13987344	13,34
Man 2mm	293	170	939	46771590	44,60
Man 1mm	586	340	1878	374172720	356,84

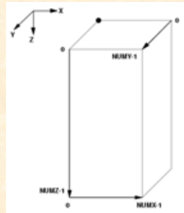


Body models: writing and reading cycles

If external cycle variable is Z, medium cycle variable is Y and internal cycle variable is X, then the file is written and read wit a cycle like

```
for(int iz = 0; iz<NUMZ; iz++)
{
        for(int iy = 0; iy<NUMY; iy++)
        {
                for(int ix = 0; ix<NUMX; ix++)
                {

                }
        }
}
```



In that case the file is organized in axial sections

Sources models: wired models and numerical integration

The source potential is related to the source currents by:

$$\nabla^2 \vec{A}_s = -\mu \cdot \vec{J}_s$$

That has the solution (considering the conductor thickness negligible):

$$\vec{A}_s(Q) = \frac{\mu I}{4\pi} \cdot \int_{Conductor} \frac{d\vec{C}}{|Q-P|}$$

The latter expression can be integrated numericaly. If a conductor is represented by a segmented line, the analitical solution exists. A current I flowing along the portion of the z axis, between $z = a$ and $z = b$ generates a vector potential in Q given by:

$$\vec{A}_s(x,y,z) = \frac{\mu I}{4\pi} \cdot \ln\left(\frac{\sqrt{x^2+y^2+(z-b)^2} - (z-b)}{\sqrt{x^2+y^2+(z-a)^2} - (z-a)} \right) \hat{i}_z$$

if $a \to -\infty$ and $b \to +\infty$

$$\vec{A}_s(x,y,z) = -\frac{\mu I}{4\pi} \cdot \ln\left(x^2+y^2\right)\hat{i}_z$$

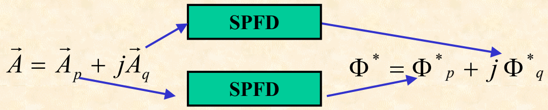
if $a = 0$ and $b \to +\infty$

$$\vec{A}_s(x,y,z) = -\frac{\mu I}{4\pi} \cdot \ln\left(\sqrt{x^2+y^2-z^2} - z\right)\hat{i}_z$$

The expression ror arbitrary oriented elements can be derived using rotation and translation

Sources model: multiphase case

Since the SPFD discretized equation has real coefficients, the excitation due to the in-phase and quadrature components of the source can be computed separately.



$$\vec{A} = \vec{A}_p + j\vec{A}_q \qquad \Phi^* = \Phi^*_p + j\,\Phi^*_q$$

The resulting solutions can then be combined at the post-processing stage to yeld the complex induced field and density current.

$$\vec{J} = \sigma\left[\vec{E}_p + j\,\vec{E}_q\right] = \sigma\left[\left(-\nabla\Phi^*_p - \omega\,\vec{A}_p\right) + j\left(-\nabla\Phi^*_q - \omega\,\vec{A}_q\right)\right]$$

That aproach is usually faster than implementing the SPFD method for complex numbers (unless is available a fast complex arithmetic tool).

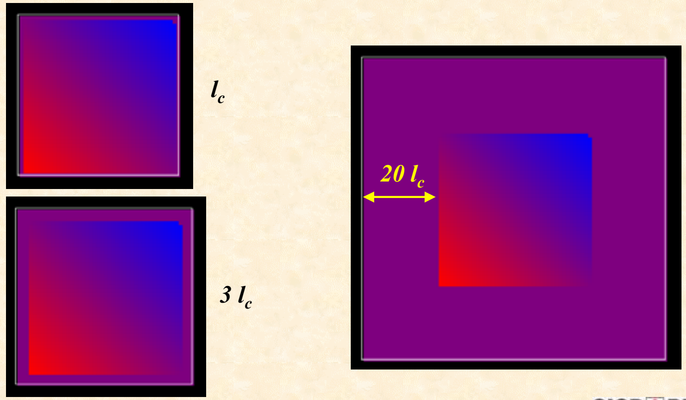
Scalar potential boundary conditions on the borders of the considered volume

$$\bar\sigma_{m1}\cdot\Phi_{n1}+\bar\sigma_{m2}\cdot\Phi_{n2}+\bar\sigma_{m3}\cdot\Phi_{n3}+\bar\sigma_{m4}\cdot\Phi_{n4}-\left(\sum_{i=1}^{4}\bar\sigma_{mi}\right)\cdot\Phi_{n0}=$$
$$=-j\omega\cdot l_c\left(\bar\sigma_{m1}A_{m1}-\bar\sigma_{m2}A_{m2}+\bar\sigma_{m3}A_{m3}-\bar\sigma_{m4}A_{m4}\right)$$

Scalar potential values in conductive regions (borders included) are not influenced by values in not conductive regions.
The body model can be incorporated in a shell of vacuum with thickness of $\mathbf{l_c}$.
The shell is added for computational reason: we don't wont to go outside the discretized domine border during iteration. Scalar potential on the border is fixed (usually to 0) and iteration stops one cell before.

Scalar potential boundary conditions on the borders of the considered volume

Omogeneous cube with σ=1,5 S/m exposed to a uniform magnetic field of 1 mT. Different shell's thickness were applied:

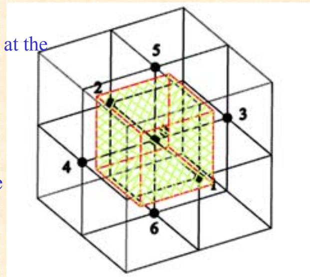


Implementation of the 3D SPFD method :memory patterns

$$\bar\sigma_{m1}\cdot\Phi_{1}+\bar\sigma_{m2}\cdot\Phi_{2}+\bar\sigma_{m3}\cdot\Phi_{3}+\bar\sigma_{m4}\cdot\Phi_{4}+\bar\sigma_{m5}\cdot\Phi_{5}+\bar\sigma_{m6}\cdot\Phi_{6}-\left(\sum_{i=1}^{6}\bar\sigma_{mi}\right)\cdot\Phi_{0}=$$
$$=-j\omega\cdot l_c\left(\bar\sigma_{m1}A_{m1}-\bar\sigma_{m2}A_{m2}+\bar\sigma_{m3}A_{m3}-\bar\sigma_{m4}A_{m4}+\bar\sigma_{m5}A_{m5}-\bar\sigma_{m6}A_{m6}\right)$$

For every node are needed:

- 7 cofficients for the scalar potential values at the nodes (can be reduced to 4 thanks to the structure of the nodes disposition)
- 1 "known term"
- 1 scalar potential value



Different approaches can be used to memorize and recall those coefficients during the SPFD problem resolution. A compromize between speed and memory occupation has usually to be founded.

Implementation of the 3D SPFD method :memory patterns

Example of memory pattern on a 32 bit platform using double precision: amount of memory for every grid node.

- 1 double for every coefficient (8 coeff. 5 if the simmetry of the matrix is taken into account and in particular that σ_{m2} is σ_{m1} for the neighbouring cell).
- 1 double for the unknown scalar potential value in the cell

48 bytes for every cell

Other patterns can be used and the number of bytes needed for every cell can be decreased to 15

Model	nx	ny	nz	n_cells	Model memory occupation [Mb]	Demanded resources[Mb]
Head 1mm	178	235	211	8826130	8,42	404
Man 3mm	196	114	626	13987344	13,34	640
Man 2mm	293	170	939	46771590	44,60	2141
Man 1mm	586	340	1878	374172720	356,84	17128

Computation time

Omogeneous conductive cube exposed to uniform magnetic field of 1 mT (100Hz)

nx	ny	nz	kcells	iterations	time [s]
42	42	42	74	230	3,89
44	44	44	85	219	4,26
48	48	48	111	198	5,08
60	60	60	216	145	10,44
65	65	65	275	122	20,42
70	70	70	343	92	22,12
75	75	75	422	109	35,9
80	80	80	512	106	42,78

VHP head (1 mm) model exposed to a uniform magnetic field of 1 mT (100 Hz) **more than 2 hours**.