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The dielectric properties of biological tissues: III. Parametric models for the dielectric spectrum of tissues

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Abstract. A parametric model was developed to describe the variation of dielectric properties of tissues as a function of frequency. The experimental spectrum from 10 Hz to 100 GHz was modelled with four dispersion regions. The development of the model was based on recently acquired data, complemented by data surveyed from the literature. The purpose is to enable the prediction of dielectric data that are in line with those contained in the vast body of literature on the subject. The analysis was carried out on a Microsoft Excel spreadsheet. Parameters are given for 17 tissue types.

1. Introduction

The dielectric properties of tissues have been characterized experimentally in the frequency range 10 Hz to 20 GHz (Gabriel *et al* 1996b). The data were shown to fall well within the vast body of literature data from a recent literature review (Gabriel *et al* 1996a). The studies were instigated by the need for such information in electromagnetic dosimetry. Dosimetry problems involve the simulation of exposure situations and the calculation of internal fields within the body. Tackling such problems requires the use of numerical techniques to solve the appropriate Maxwell equations. To facilitate the incorporation of the dielectric data in such procedures, it is convenient to express their frequency dependence as parametric expressions to provide access to data at all frequencies of interest. Examples of such models have been reported by Foster *et al* (1979) for brain tissue and by Schepps and Foster (1980) for tumour tissue. The range of applicability of the models was limited by the data available to above 1 MHz. A similar, but more extensive analysis was carried out by Hurt (1985), who reviewed the data for muscle and described its dielectric behaviour from 10 Hz to 10 GHz in terms of five dispersion regions; the frequency dependence within each dispersion was expressed in terms of the well-known Debye model. Muscle, being one of the most widely reported tissues, was a good candidate for such an analysis.

The availability of new dielectric data over a wide frequency range enabled us to extend the multidispersion model to other tissues. The analysis presented in this paper is based on the previously reported experimental data, complemented by the data surveyed from the literature. The spectrum from 10 Hz to 100 GHz was modelled to four dispersion regions. The frequency dependence within each region was expressed as a Cole–Cole term. Results for high and low water-content tissues are reported to illustrate the analysis. In practice, the model can be used at all frequencies in the specified range.

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When applied to a subsection of the data, over the frequency range of a well characterized dispersion region, such an analysis can lead to insights into the relationship between dielectric and molecular parameters of biological materials. An example is given of a comparative study of tissue water as characterized by the dielectric parameters.

2. Model for the dielectric spectrum of a tissue

The main features of the dielectric spectrum of tissues are well known and have been reviewed and reported by Foster and Schwan (1989), to name one of the more comprehensive articles on this matter. The dielectric spectrum of a tissue is characterized by three main relaxation regions α, β and γ at low, medium and high frequencies, and other minor dispersions such as the often reported δ dispersion. In its simplest form, each of these relaxation regions is the manifestation of a polarization mechanism characterized by a single time constant, τ which, to a first order approximation, gives the following expression for the complex relative permittivity ($\hat{\varepsilon}$) as a function of angular frequency (ω):

$$\hat{\varepsilon} = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + \mathrm{j}\omega\tau}. \tag{1}$$

This is the well-known Debye expression in which ε_∞ is the permittivity at field frequencies where $\omega\tau \gg 1$, ε_s the permittivity at $\omega\tau \ll 1$, and $\mathrm{j}^2 = -1$. The magnitude of the dispersion is described as $\Delta\varepsilon = \varepsilon_s - \varepsilon_\infty$.

Hurt (1985) modelled the dielectric spectrum of muscle to the summation of five Debye dispersions in addition to a conductivity term in which σ_i is the static ionic conductivity and ε_0 is the permittivity of free space:

$$\hat{\varepsilon}(\omega) = \varepsilon_\infty + \sum_{n=1}^{5} \frac{\Delta\varepsilon_n}{1 + \mathrm{j}\omega\tau_n} + \frac{\sigma_i}{\mathrm{j}\omega\varepsilon_0}. \tag{2}$$

However, the complexity of both the structure and composition of biological material is such that each dispersion region may be broadened by multiple contributions to it. The broadening of the dispersion could be empirically accounted for by introducing a distribution parameter, thus giving an alternative to the Debye equation known as the Cole–Cole equation

$$\hat{\varepsilon}(\omega) = \varepsilon_\infty + \frac{\Delta\varepsilon}{1 + (\mathrm{j}\omega\tau)^{(1-\alpha)}} \tag{3}$$

where the distribution parameter, α, is a measure of the broadening of the dispersion. The spectrum of a tissue may therefore be more appropriately described in terms of multiple Cole–Cole dispersion

$$\hat{\varepsilon}(\omega) = \varepsilon_\infty + \sum_{n} \frac{\Delta\varepsilon_n}{1 + (\mathrm{j}\omega\tau_n)^{(1-\alpha_n)}} + \frac{\sigma_i}{\mathrm{j}\omega\varepsilon_0} \tag{4}$$

which, with a choice of parameters appropriate to each tissue, can be used to predict the dielectric behaviour over the desired frequency range.

3. Procedure to parametrize the data

We are now required to determine the parameters of equation (4) that will fit the dielectric data. Numerical least-squares minimization techniques are the most common approach to obtain the best estimate of the parameters and the confidence interval associated with them (Grant *et al* 1978). This procedure is not appropriate in the present situation for

several reasons. First, the data to be fitted span several orders of magnitude, creating a bias towards fitting the low frequency data and rendering the fit insensitive to the high frequency parameters. Moreover, the parameters of the model are intercorrelated to the extent that there is no unique solution. While these barriers are not insurmountable, a different approach was nonetheless sought.

The analysis was carried out using a Microsoft Excel spreadsheet. The data for each tissue were compiled in a workbook made up of several pages of data, calculations and graphical representations. The data sheet was configured to accept up to 22 sets of data, gathered from the literature (Gabriel *et al* 1996a) as well as experimental data from a recent study (Gabriel *et al* 1996b). The model was programmed into a sheet and, in conjunction with the collated data, a graphical representation was generated. The graphs produced displayed the full range of experimental and 'literature' data, and a representation of the fitting equation generated from the parameter list. The computer workstation used allowed the graphical representation to be displayed on a single screen while the the parameter list, fitting function and other plotted data were displayed on further screens. This configuration allowed for continuous monitoring of the all the variables and their contribution in the various stages of the fitting process. A systematic procedure was then followed in which the main parameters of the model were fitted, going from high to low frequencies, keeping the α values at zero in the first instance. Successive refinements were then made in a similar manner but including all parameters. The value for ε_∞ was fixed at 2.5 or 4 for low and high water-content tissues respectively. These values are consistent with the prevailing knowledge of aqueous mixture behaviour. Three main dispersions are evident in the spectrum extending from 10 Hz to 100 GHz, but a four Cole–Cole model provided more flexibility to achieve a better fit to the data, and was therefore adopted throughout the study.

A particular feature of the working graph is that each successive summation of the Cole–Cole model is plotted. This emphasizes the contribution of each dispersion to the final model and helps with the adjustment of the fitting parameters. The whole fitting process is visual and requires an understanding of the Cole–Cole function and an appreciation of the correlation between parameters. The fitting procedure is terminated when positive and negative changes to the parameters produce no visible difference. The resulting model represents a good fit to the data rather then a unique solution based on a mathematical argument.

4. Results

Figure 1(a)–(q) shows a graphical prediction of the model applied to 17 tissues together with the corresponding literature data. The model parameters used to generate the graphs in figure 1 are given in table 1.

The main purpose of this analysis is the prediction of dielectric data that are in line with those contained in the vast body of literature on the subject. It should, however, be noted that, in view of the nature of the model, the result obtained for each spectrum is not unique. Consequently, taken as a whole this model should not be used to correlate the dielectric parameters to the structure and composition of the various tissues. This can be done by examining, in a comparative manner, parts of the spectrum and the underlying mechanism responsible for it.

For example, at frequencies in excess of a few hundred MHz, the dipolar orientation of the water molecules is the dominant polarization mechanism. The frequency dependence of

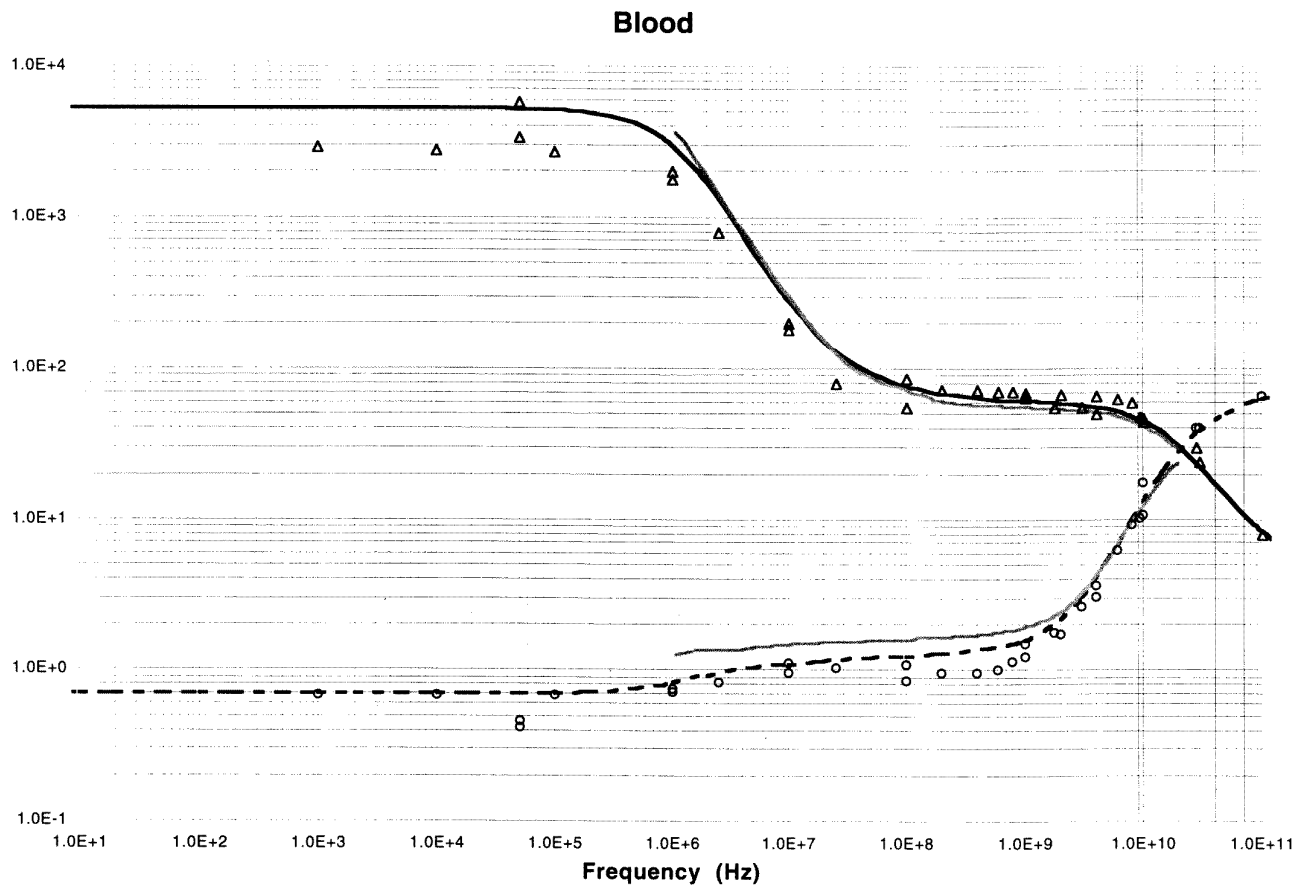

Figure 1. Permittivity and conductivity of tissues: prediction of the model (black filled and dotted curves), experimental data at 37 °C (grey filled and dotted curves) and data from the literature (triangles and circles). (*a*) Blood.

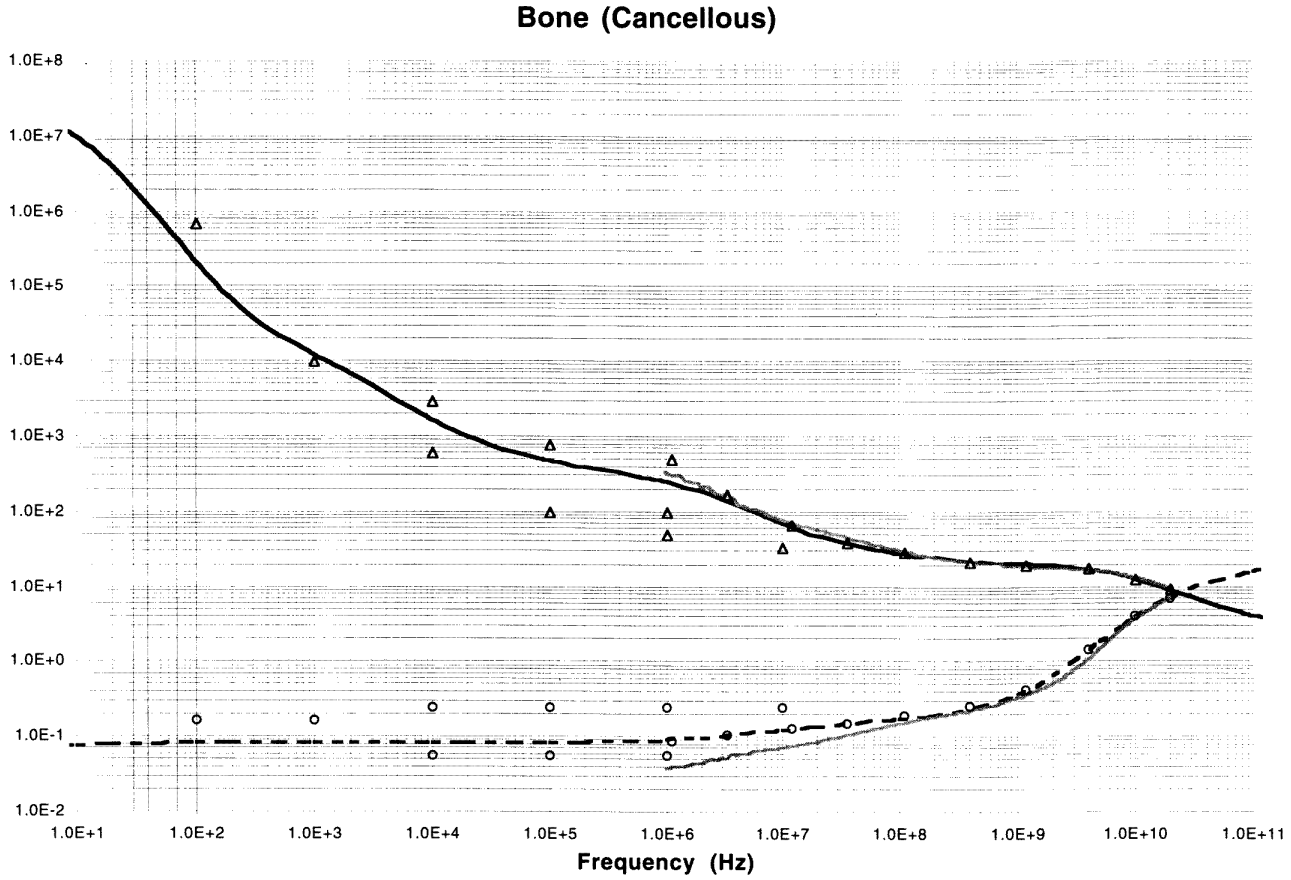

Figure 1. (*b*) Bone (cancellous).

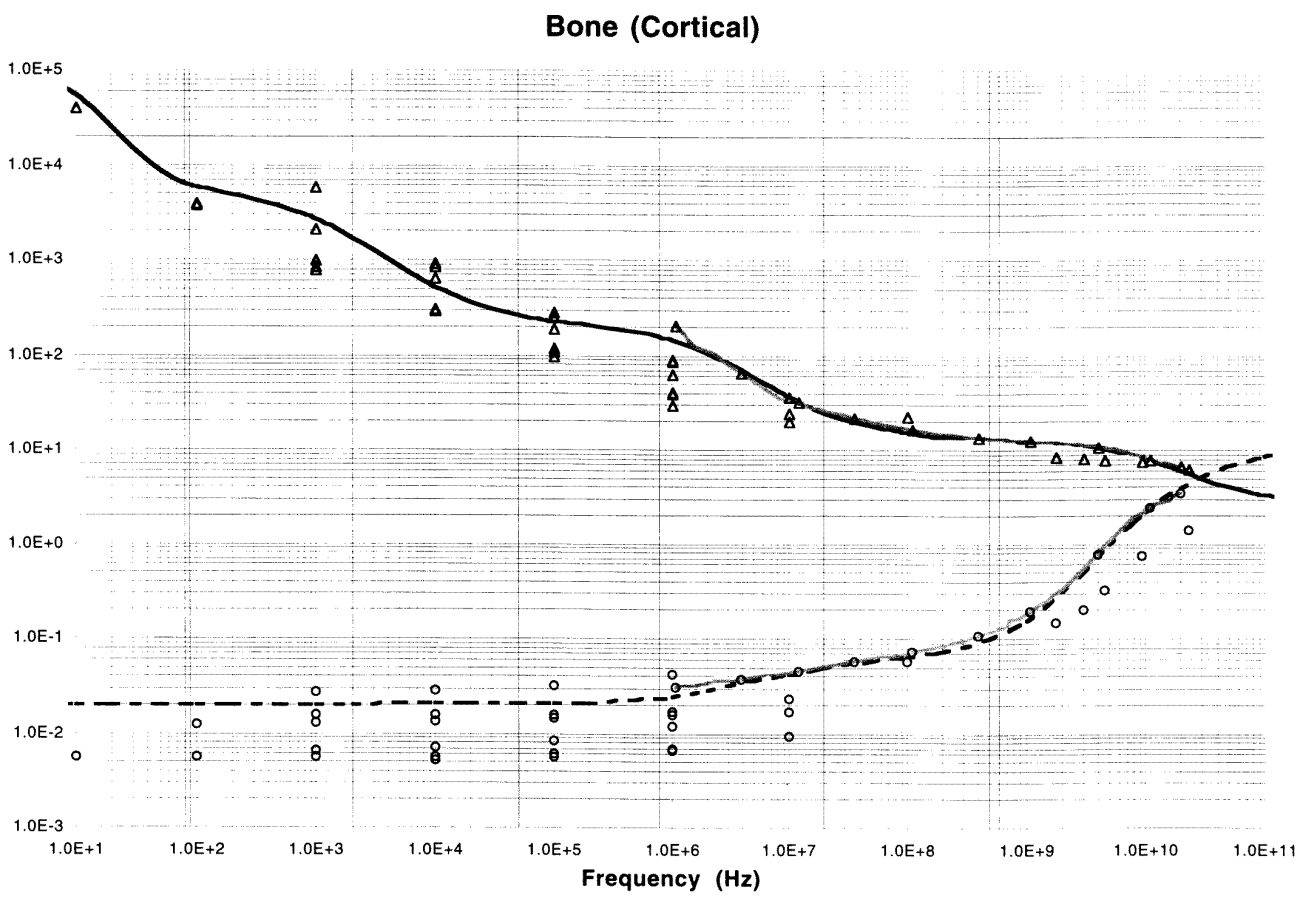

Figure 1. (*c*) Bone (cortical).

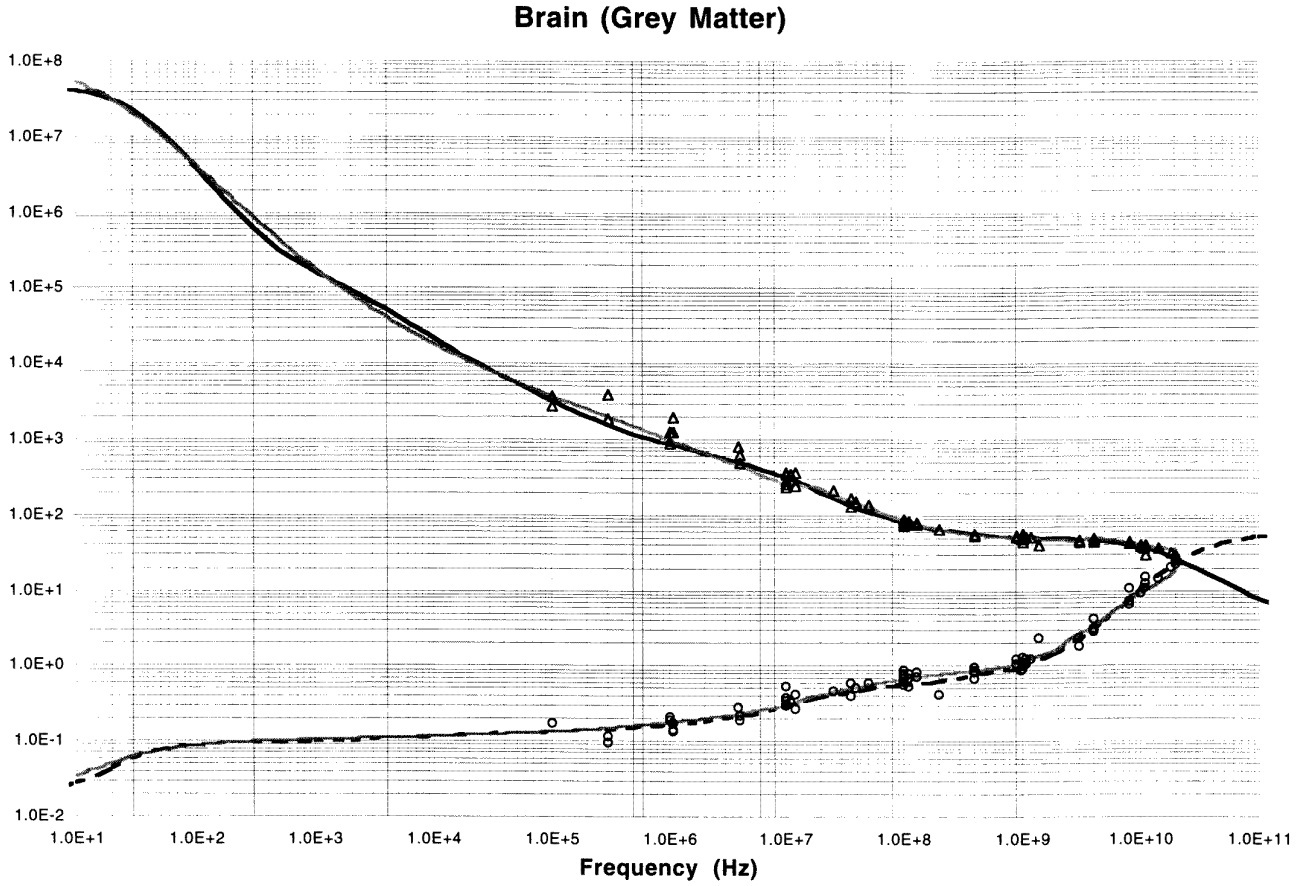

Figure 1. (*d*) Brain (grey matter).

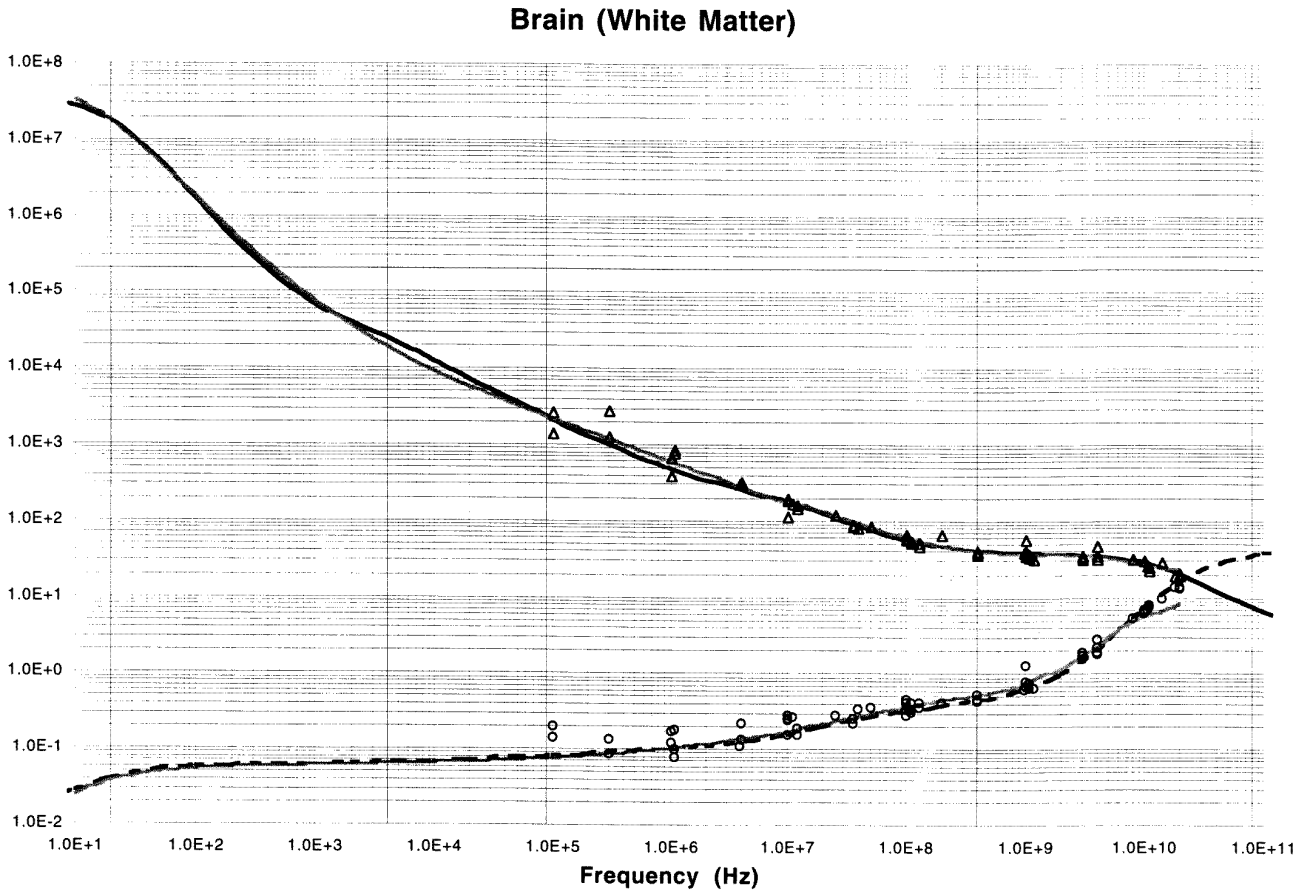

Figure 1. (*e*) Brain (white matter).

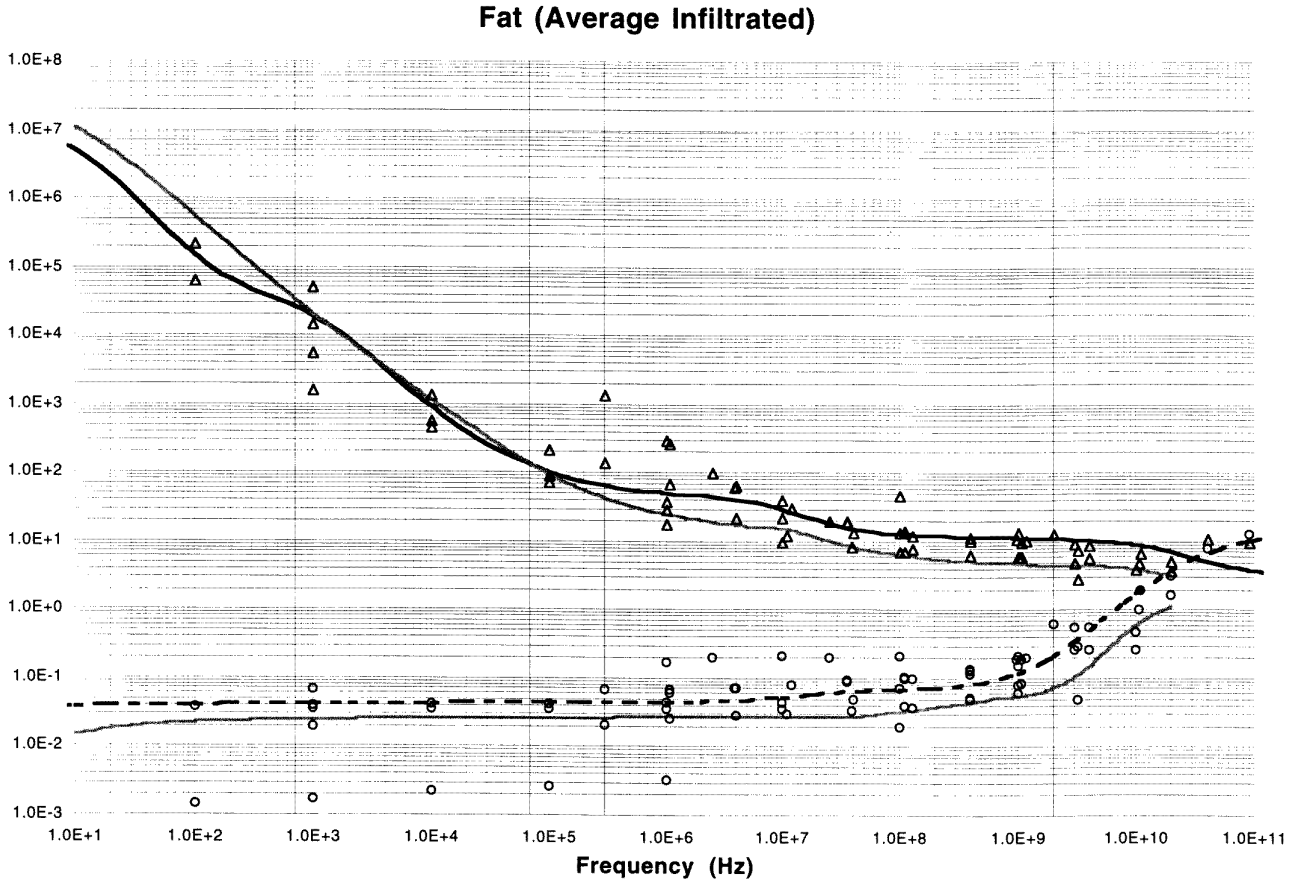

Figure 1. (*f*) Fat (infiltrated).

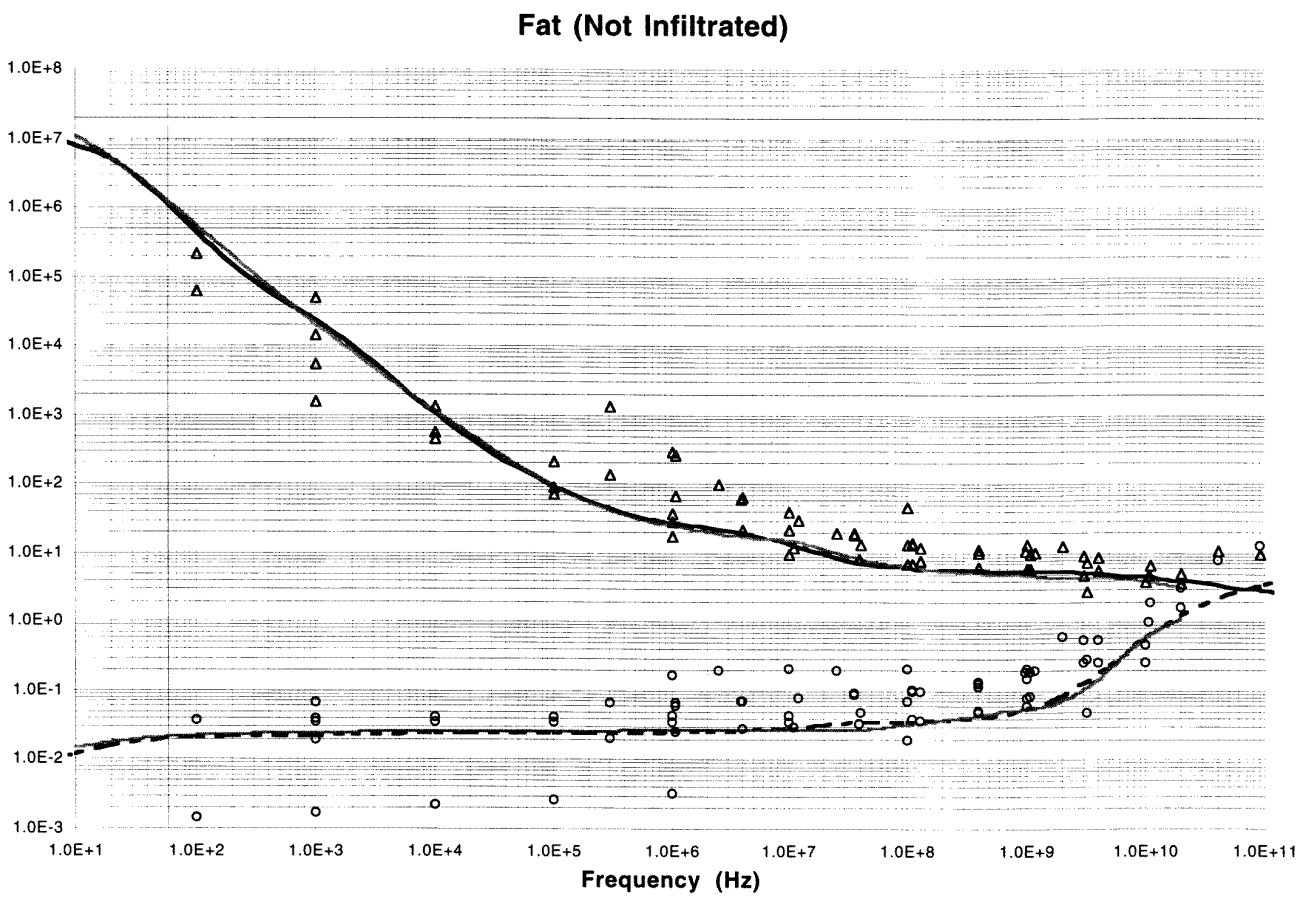

Figure 1. (*g*) Fat (not infiltrated).

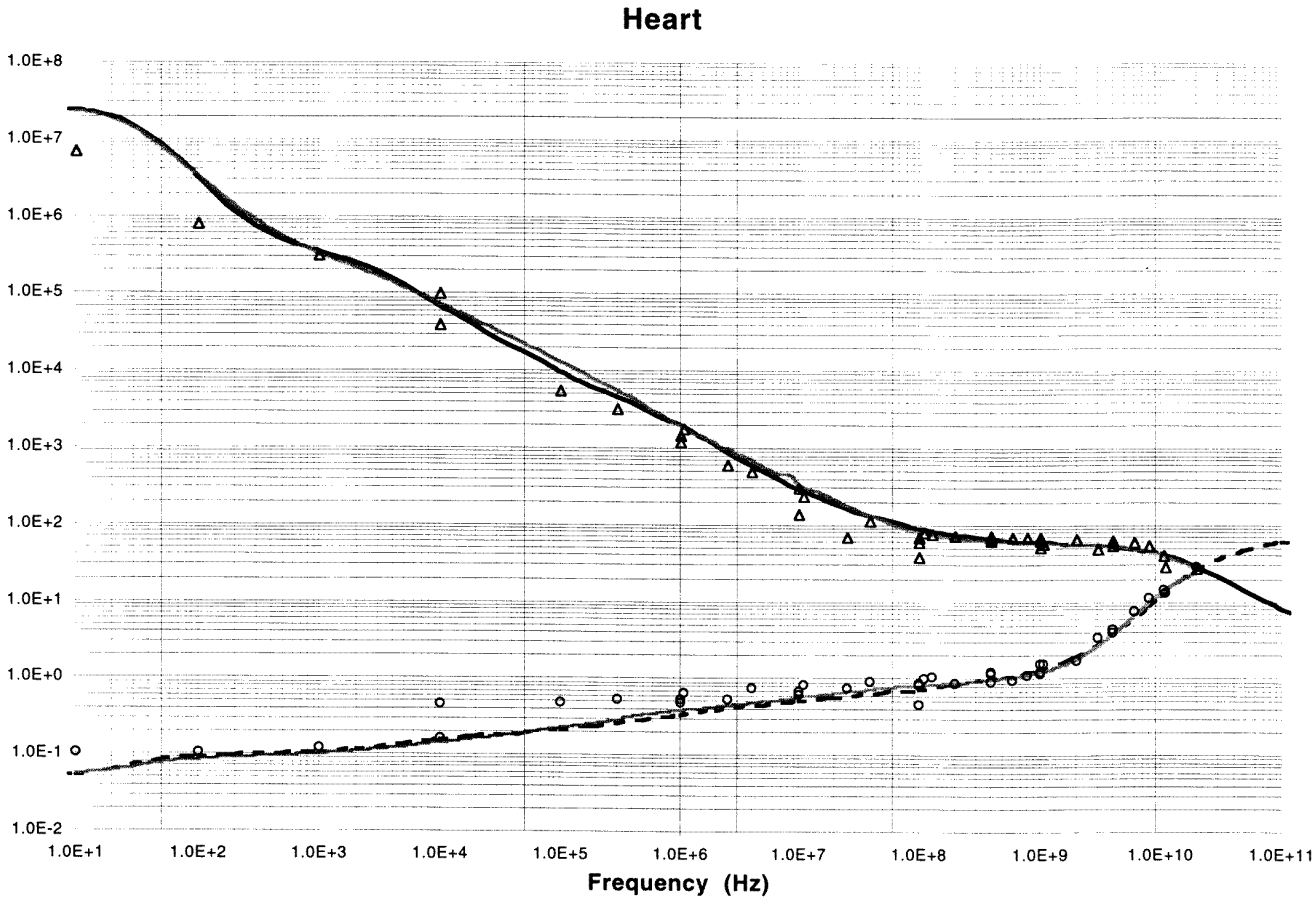

Figure 1. (*h*) Heart.

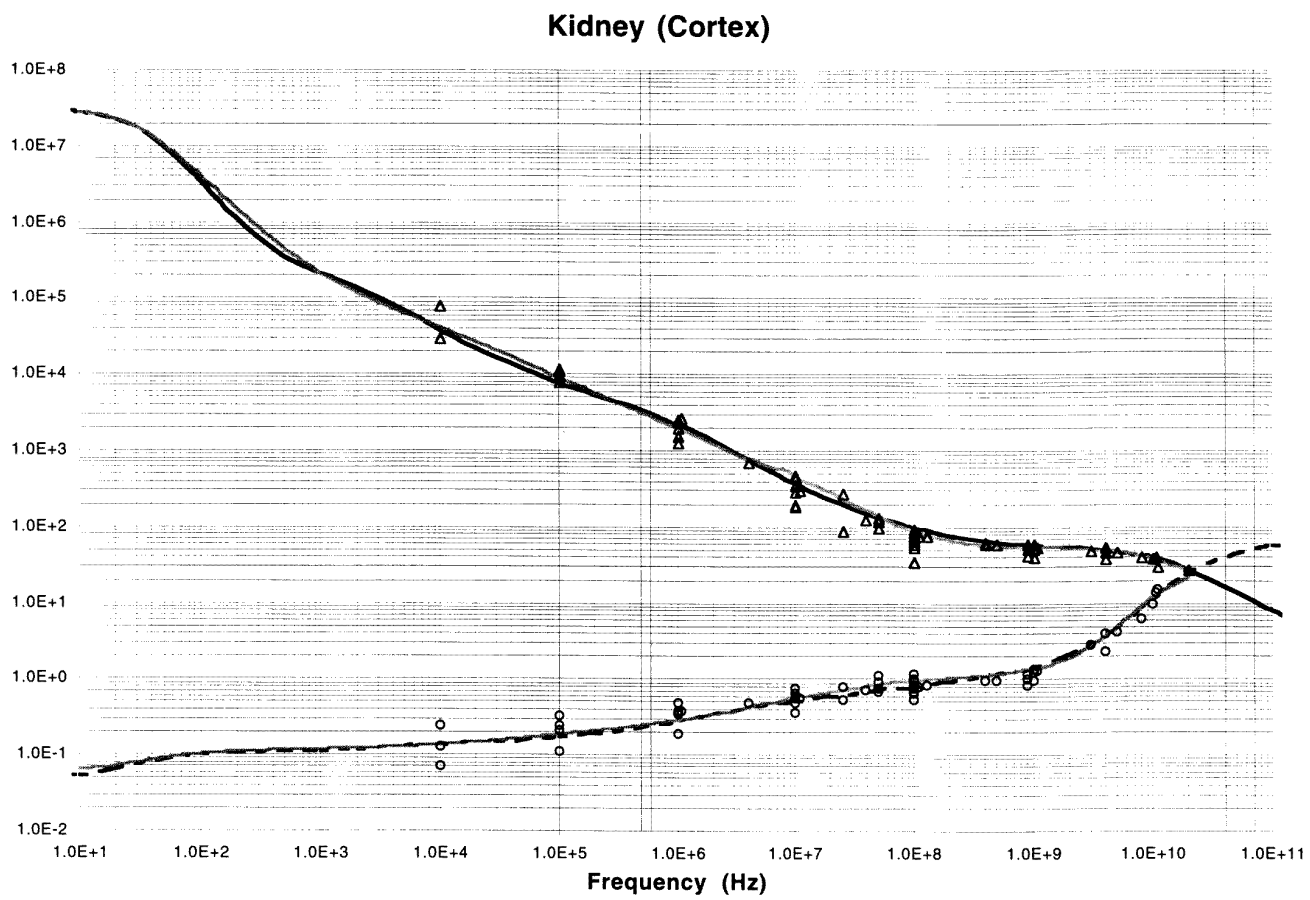

Figure 1. (*i*) Kidney (cortex).

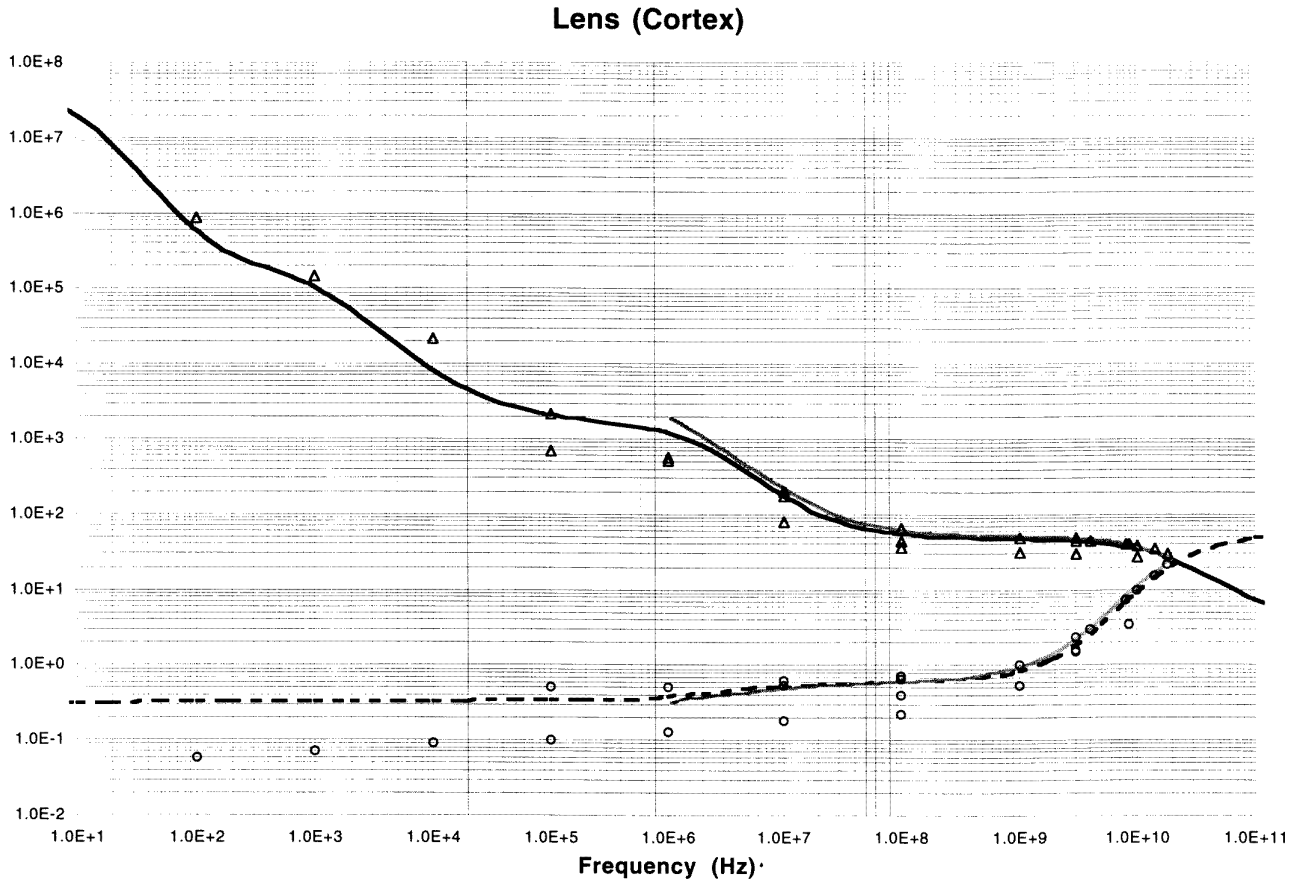

Figure 1. (*j*) Lens (cortex).

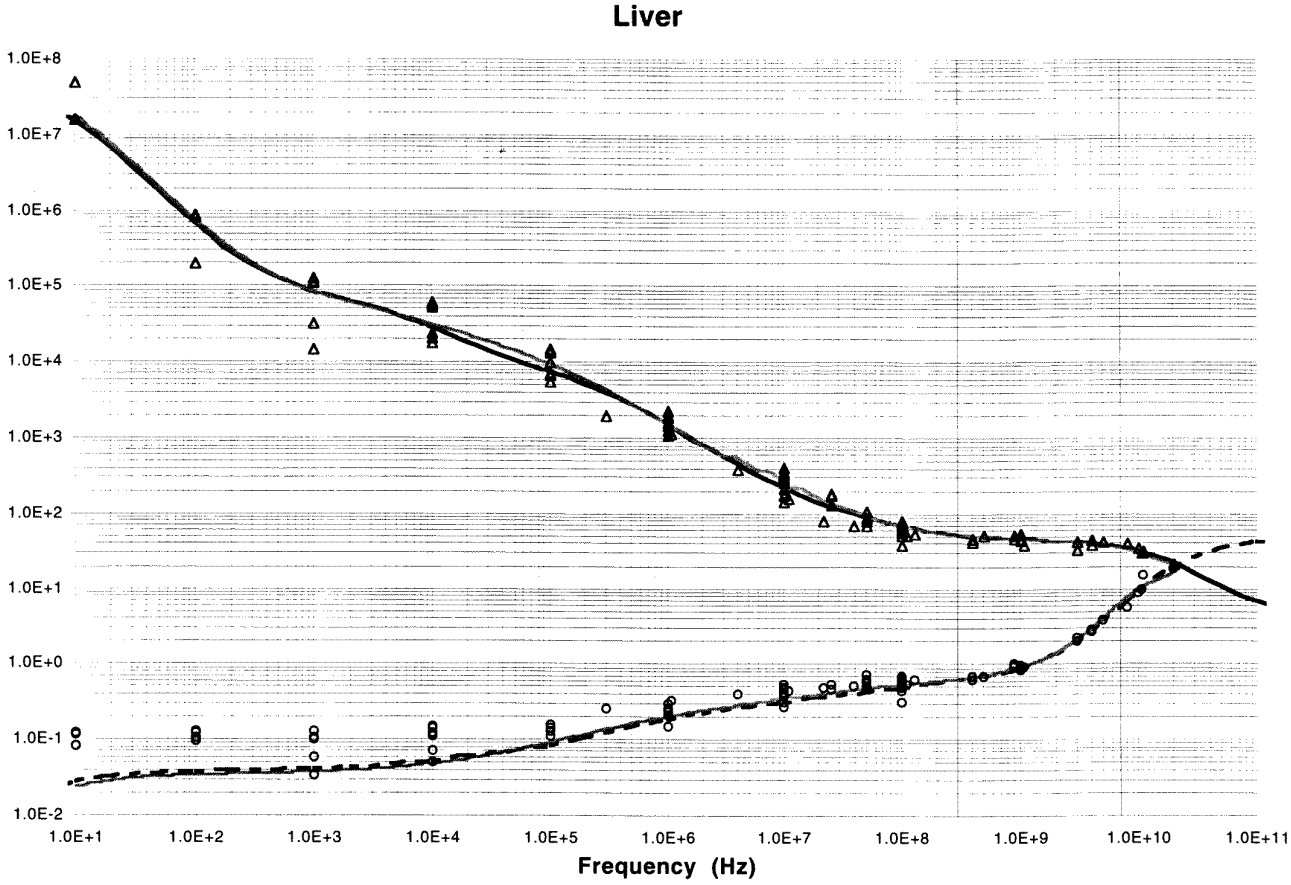

Figure 1. (k) Liver.

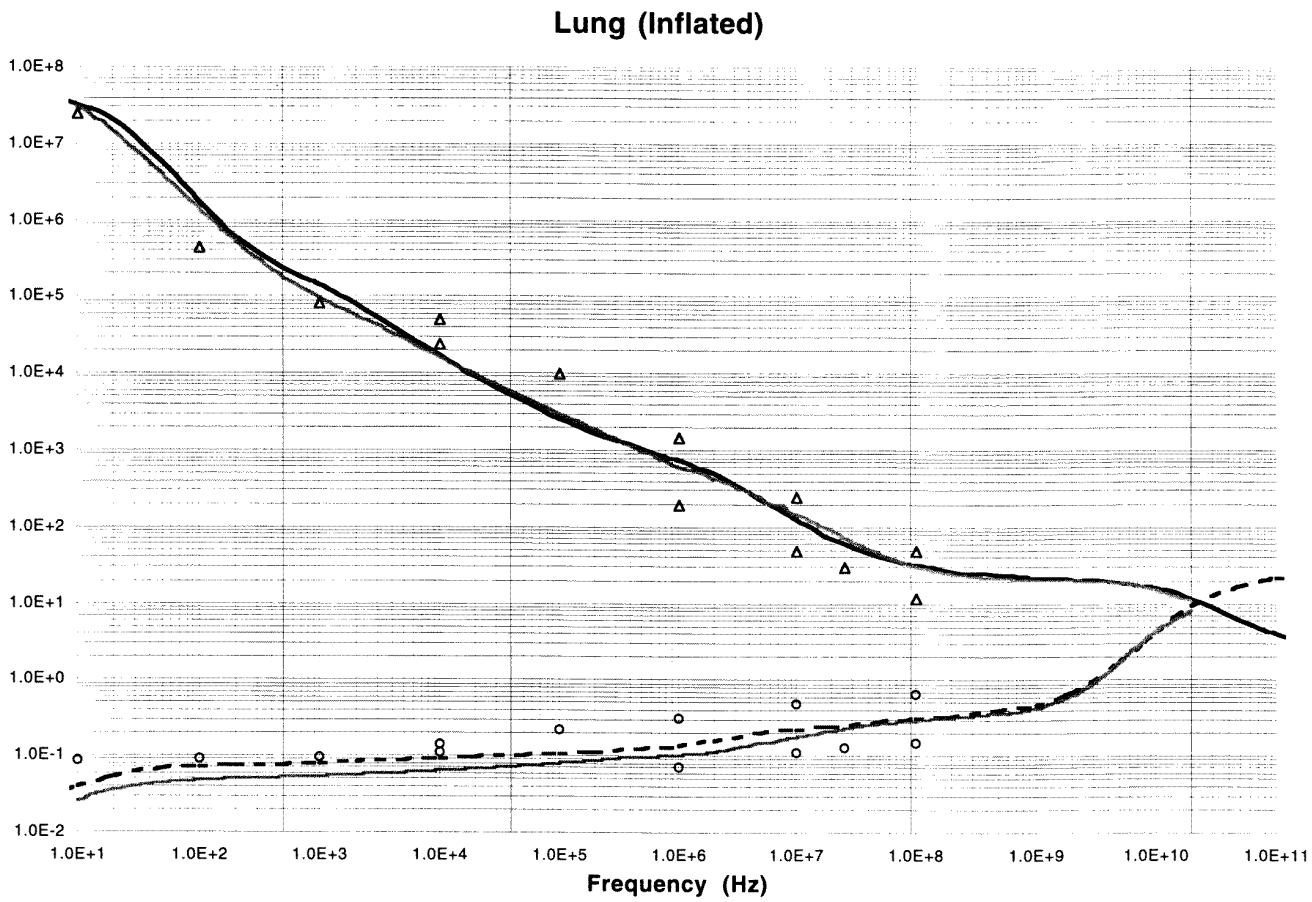

Figure 1. (*l*) Lung (inflated).

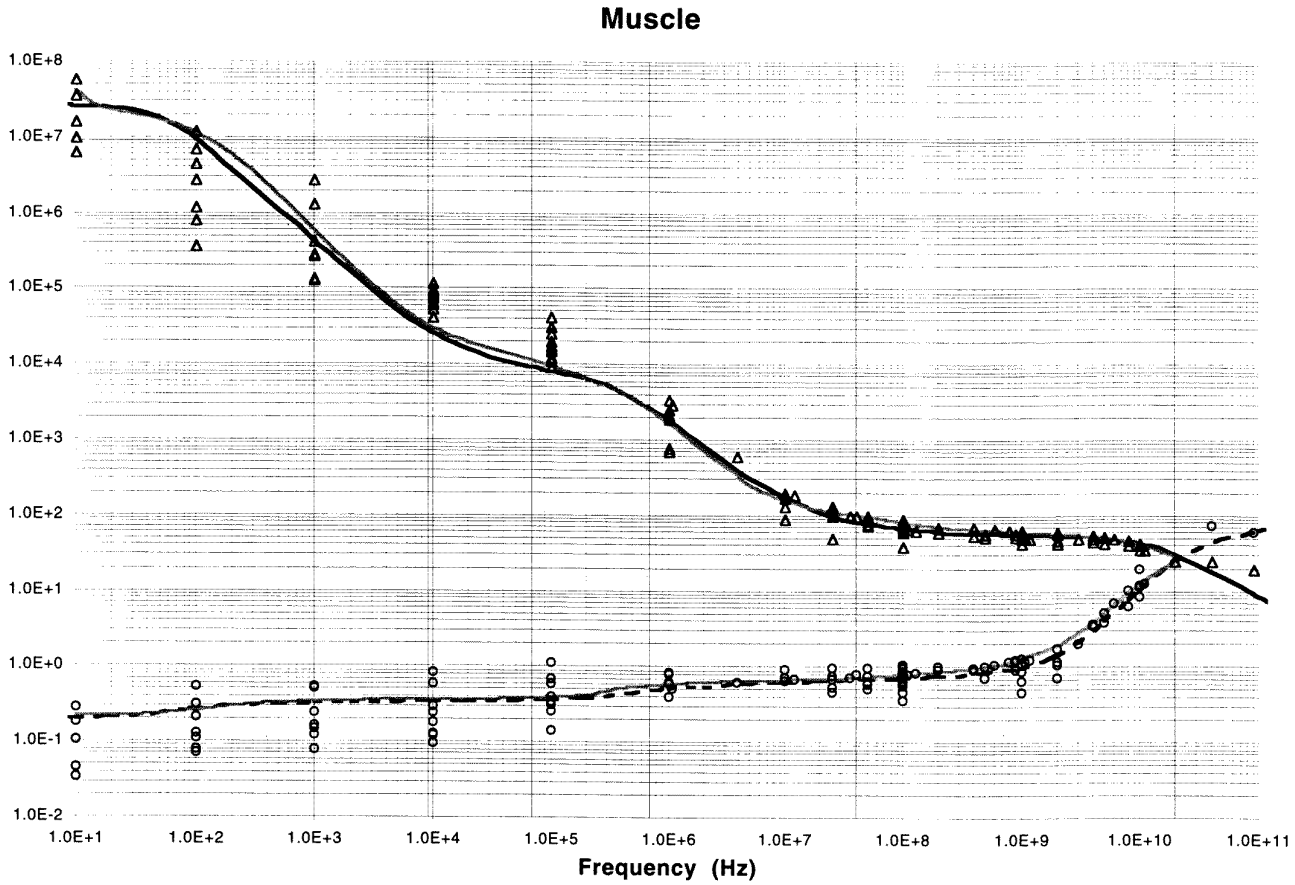

Figure 1. (*m*) Muscle.

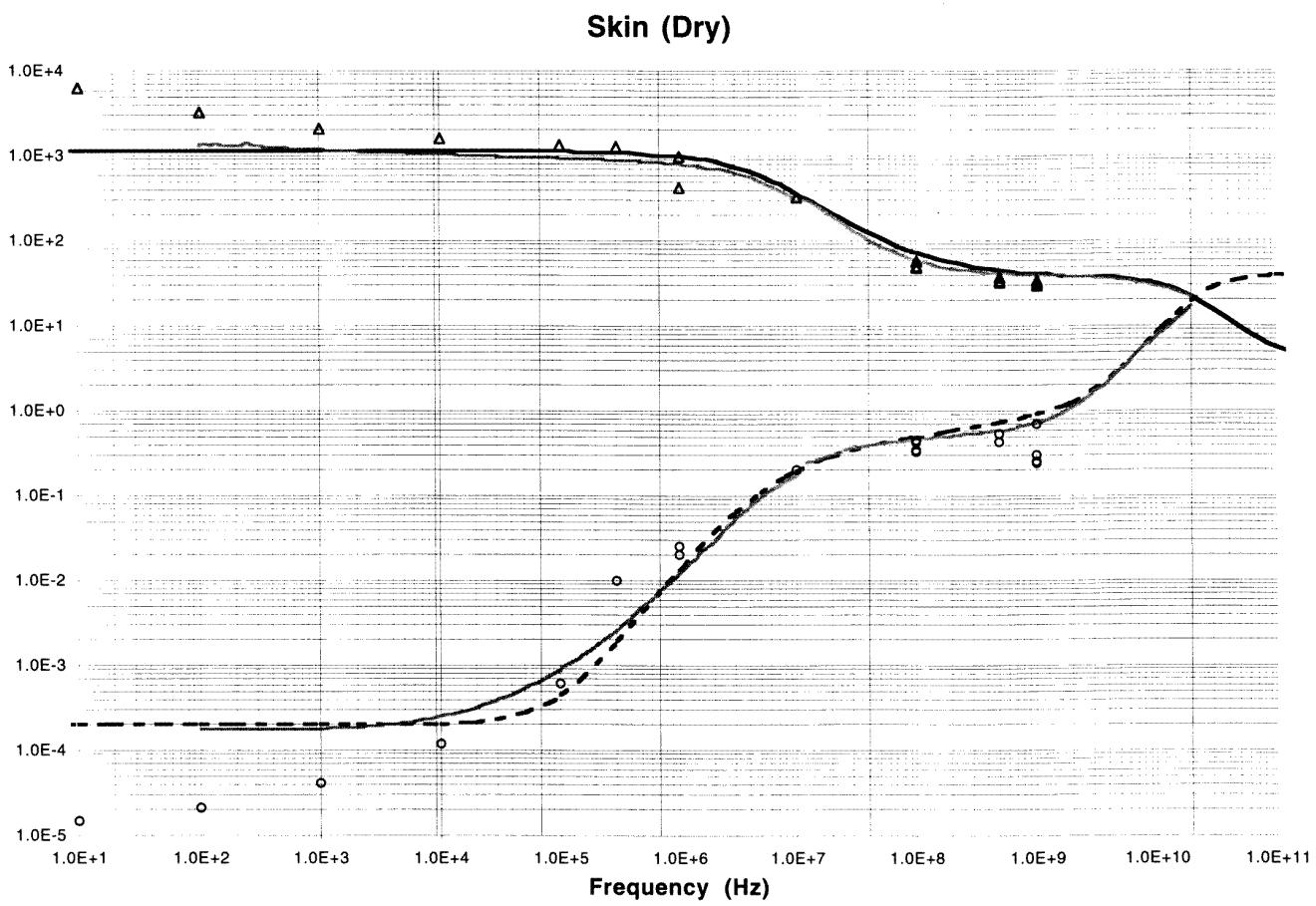

Figure 1. (*n*) Skin (dry).

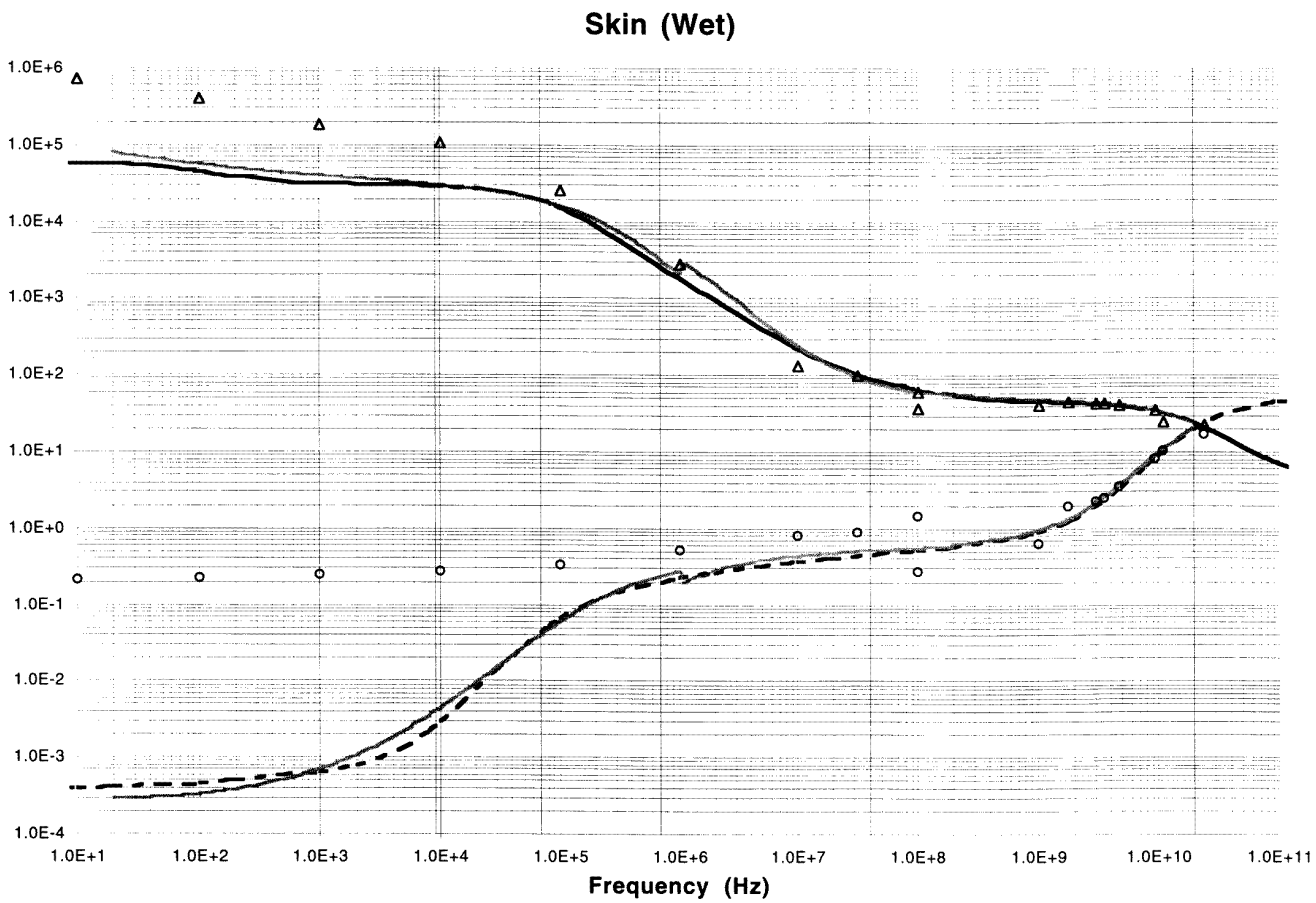

Figure 1. (*o*) Skin (wet).

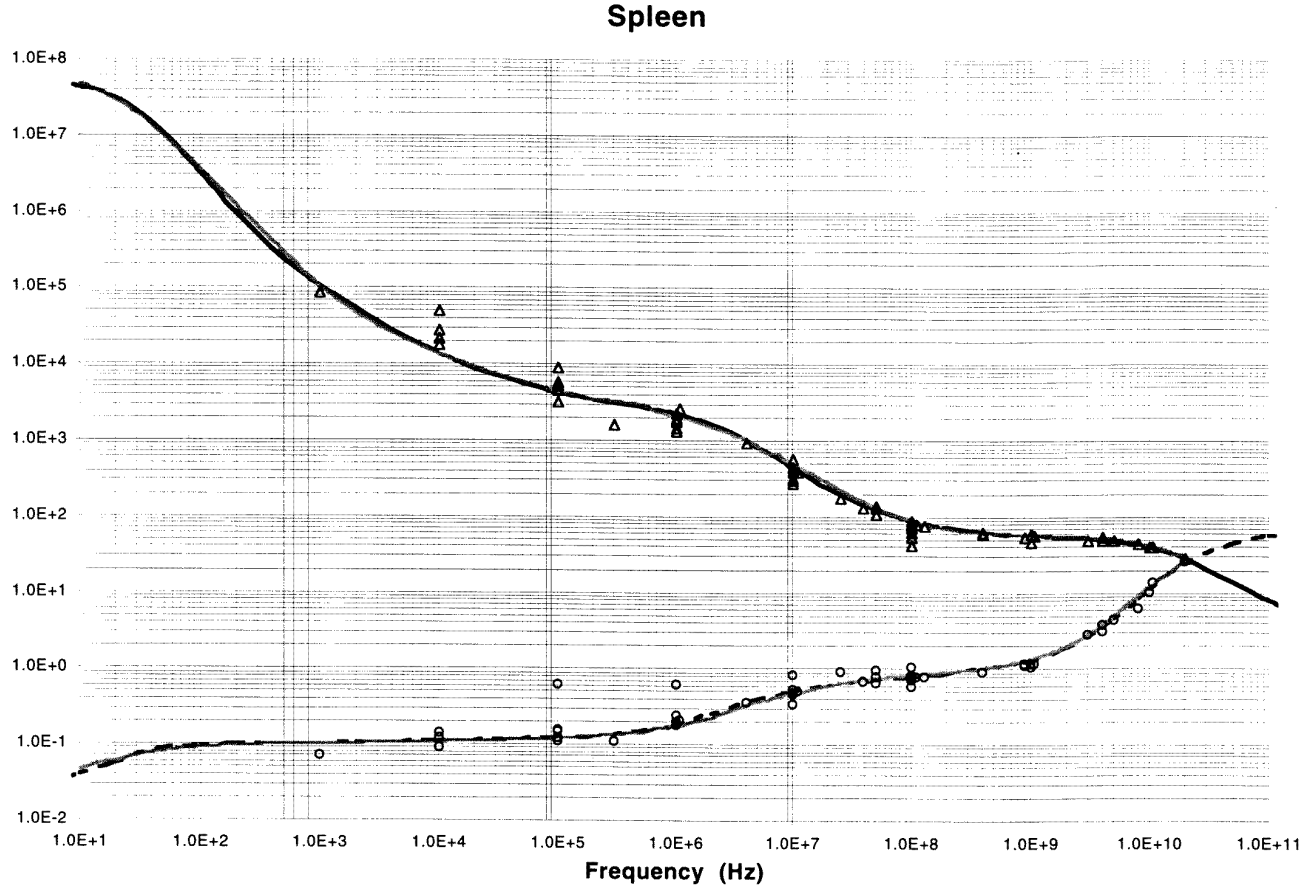

Figure 1. (*p*) Spleen.

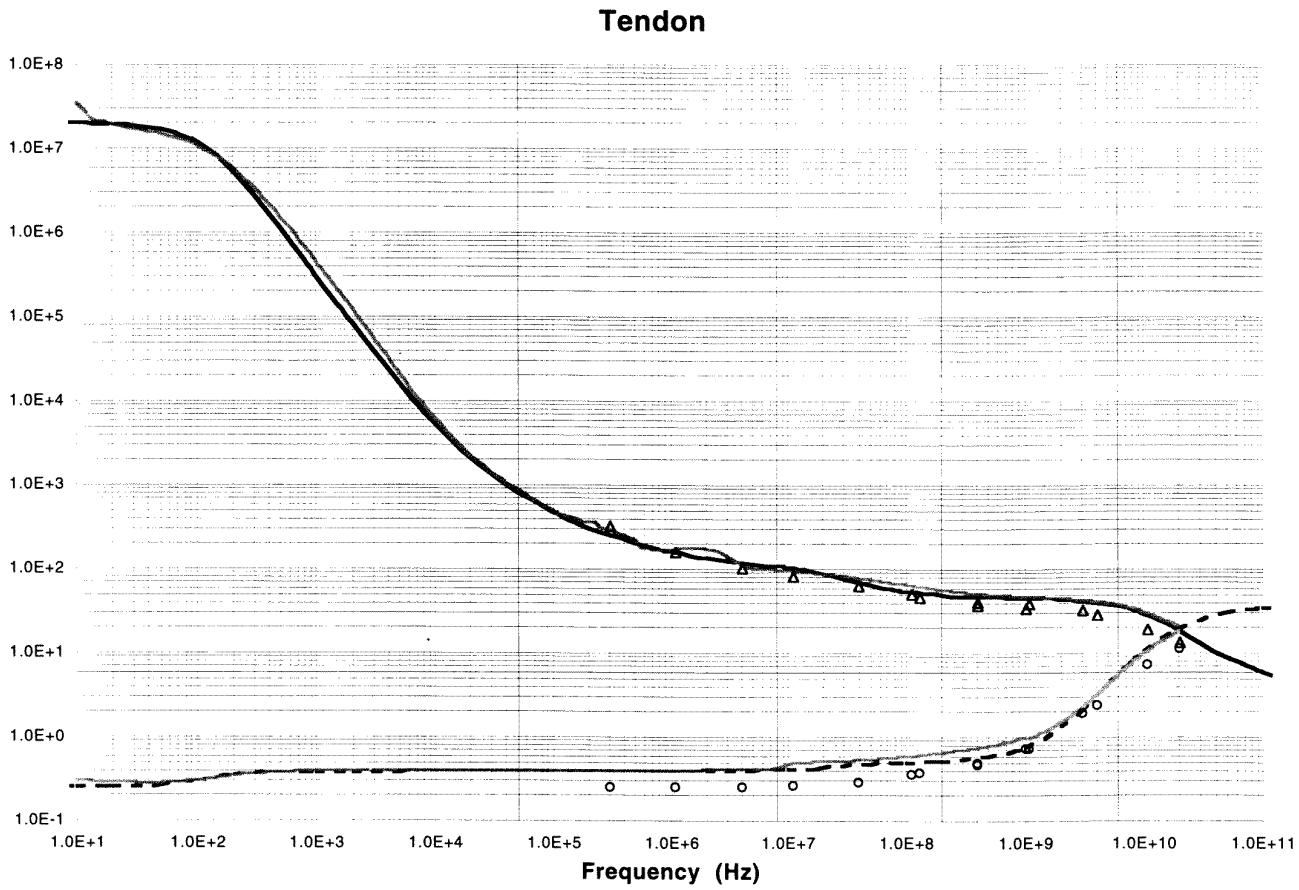

Figure 1. (q) Tendon.

Table 1. Parameters of equation (4) used to predict the dielectric properties of tissues in figures 1(*a*)–(*q*).

Tissue type	ε_∞	$\Delta\varepsilon_1$	τ_1 (ps)	α_1	$\Delta\varepsilon_2$	τ_2 (ns)	α_2	$\Delta\varepsilon_3$	τ_3 (μs)	α_3	$\Delta\varepsilon_4$	τ_4 (ms)	α_4	σ
Blood	4.0	56.0	8.38	0.10	5200	132.63	0.10	0.0			0.0			0.7000
Bone (cancellous)	2.5	18.0	13.26	0.22	300	79.58	0.25	2.0×10^4	159.15	0.20	2.0×10^7	15.915	0.00	0.0700
Bone (cortical)	2.5	10.0	13.26	0.20	180	79.58	0.20	5.0×10^3	159.15	0.20	1.0×10^5	15.915	0.00	0.0200
Brain (grey matter)	4.0	45.0	7.96	0.10	400	15.92	0.15	2.0×10^5	106.10	0.22	4.5×10^7	5.305	0.00	0.0200
Brain (white matter)	4.0	32.0	7.96	0.10	100	7.96	0.10	4.0×10^4	53.05	0.30	3.5×10^7	7.958	0.02	0.0200
Fat (infiltrated)	2.5	9.0	7.96	0.20	35	15.92	0.10	3.3×10^4	159.15	0.05	1.0×10^7	15.915	0.01	0.0350
Fat (not infiltrated)	2.5	3.0	7.96	0.20	15	15.92	0.10	3.3×10^4	159.15	0.05	1.0×10^7	7.958	0.01	0.0100
Heart	4.0	50.0	7.96	0.10	1200	159.15	0.05	4.5×10^5	72.34	0.22	2.5×10^7	4.547	0.00	0.0500
Kidney	4.0	47.0	7.96	0.10	3500	198.94	0.22	2.5×10^5	79.58	0.22	3.0×10^7	4.547	0.00	0.0500
Lens cortex	4.0	42.0	7.96	0.10	1500	79.58	0.10	2.0×10^5	159.15	0.10	4.0×10^7	15.915	0.00	0.3000
Liver	4.0	39.0	8.84	0.10	6000	530.52	0.20	5.0×10^4	22.74	0.20	3.0×10^7	15.915	0.05	0.0200
Lung (inflated)	2.5	18.0	7.96	0.10	500	63.66	0.10	2.5×10^5	159.15	0.20	4.0×10^7	7.958	0.00	0.0300
Muscle	4.0	50.0	7.23	0.10	7000	353.68	0.10	1.2×10^6	318.31	0.10	2.5×10^7	2.274	0.00	0.2000
Skin (dry)	4.0	32.0	7.23	0.00	1100	32.48	0.20	0.0			0.0			0.0002
Skin (wet)	4.0	39.0	7.96	0.10	280	79.58	0.00	3.0×10^4	1.59	0.16	3.0×10^4	1.592	0.20	0.0004
Spleen	4.0	48.0	7.96	0.10	2500	63.66	0.15	2.0×10^5	265.26	0.25	5.0×10^7	6.366	0.00	0.0300
Tendon	4.0	42.0	12.24	0.10	60	6.37	0.10	6.0×10^4	318.31	0.22	2.0×10^7	1.326	0.00	0.2500

the complex permittivity may be expressed as

$$\hat{\varepsilon}(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + (\mathrm{j}\omega\tau)^{1-\alpha}} + \frac{\sigma}{\mathrm{j}\omega\varepsilon_0}. \tag{5}$$

All the parameters are as in equation (3), and σ is the conductivity due to ionic drift and the lower frequency polarization mechanisms. When the high frequency parts of the spectrum are fitted to (5) using a least-squares minimization procedure, the dispersion parameters (table 2) may be used to gain an insight into the dielectric response of tissue water. The analysis was applied in the frequency range above 400 MHz to reflect the response of all tissue water. The water content of the tissues considered ranged from $> 95\%$ for vitreous humour and $> 85\%$ for retina, to $< 20\%$ for cortical bone.

Table 2. Dielectric parameters of water dispersion in tissues obtained by analysis of the experimental results at 37 °C. The Δ terms correspond to the 95% confidence interval.

Tissue	ε_s	$\Delta\varepsilon_s$	τ (ps)	$\Delta\tau$ (ps)	α	$\Delta\alpha$	σ (S m^{-1})	$\Delta\sigma$ (S m^{-1})
Bone (cortex)	14.9	0.16	13.8	0.48	0.26	0.01	0.092	0.005
Bone (section)	22.1	0.17	14.4	0.33	0.22	0.01	0.208	0.005
Cartilage	43.6	0.63	12.8	0.55	0.27	0.02	0.58	0.02
Cornea	53.0	0.45	8.72	0.17	0.13	0.01	1.05	0.02
Lens (cortex)	52.1	0.32	9.18	0.16	0.11	0.01	0.72	0.01
Lens (nucleus)	38.1	0.26	11.3	0.22	0.20	0.01	0.33	0.01
Retina	67.3	0.33	7.25	0.08	0.05	0.01	1.42	0.02
Brain (grey)	55.5	0.50	7.76	0.15	0.12	0.02	1.03	0.02
Brain (white)	37.0	0.29	8.04	0.21	0.24	0.01	0.47	0.01
Cerebellum	50.2	0.41	8.52	0.21	0.09	0.02	0.89	0.02
Dura	49.2	0.46	9.63	0.26	0.14	0.02	0.77	0.02
Brain stem	34.6	0.26	8.45	0.21	0.20	0.01	0.47	0.01
Tongue (*in vivo*)	57.7	0.43	9.12	0.20	0.08	0.01	0.63	0.02
Aqueous humour	74.2	0.30	6.81	0.08	0.01	0.01	1.83	0.01
Water	74.1		6.2		0.0		> 0.0001	

The correlation between ε_s and tissue water content is an obvious and expected result. The value of the distribution parameter (α) is significant for most tissues and negligible for body fluids (as exemplified by aqueous humour). The mean relaxation time (τ) is generally longer than the value for water, indicating a restriction in the rotational ability of at least some of the tissue water molecules due to the organic environment. This effect is not manifested in body fluids in view of their low organic content (table 2). The lengthening of the relaxation time for water in biological material is a well studied effect; it is common to most organic solutes and is known to increase with solute concentration (Grant *et al* 1978, Bateman *et al* 1990). The effect has also previously been observed in tissues (Gabriel *et al* 1983).

5. Comments and conclusions

A model simulating four Cole–Cole type dispersions has been used to describe the frequency dependence of the dielectric properties in the frequency range from Hz to GHz. The predictions of the model can be used with confidence for frequencies above 1 MHz. At lower frequencies, where the literature values are scarce and have larger than average uncertainties, the model should be used with caution in the knowledge that it provides a

'best estimate' based on present knowledge. It is important to be aware of the limitations of the model—particularly where there are no data to support its predictions.

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