Technical note

Further precautions in the use of time-domain dielectric spectroscopy with biological and other lossy dielectrics

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1 Introduction

TIME-domain dielectric spectroscopy (TDDS) has been used to obtain rapidly the extremely-low-frequency (ELF) electrical properties of biological materials *in vivo* (SINGH *et al.*, 1979; HART, 1983*a*; *b*). A voltage step is applied to animal or plant tissue. The resulting current is Fourier transformed to yield the variation of capacitance, dielectric loss and conductance with frequency (dielectric spectrum). Certain precautions must be taken in the design of the measuring circuit to prevent artefacts entering the transformed spectra (HART, 1982). The present note points out other precautions which must be used in the transformation of the current for lossy dielectrics to prevent artefacts from entering in this part of the procedure.

The errors associated with time-domain techniques at high frequencies have been discussed elsewhere (DAWKINS *et al.*, 1981). At high frequencies a reflected waveform is sampled and transformed, whereas in ELF studies a polarisation or depolarisation current passing through the material is analysed. Use of a finite-time window introduces errors in both methods. The very slow decay in discrete steps of the polarisation current for lossy materials, however, leads to special problems in using the transformation to obtain ELF dielectric properties.

The application of a voltage step to a lossy dielectric results in the passage of a current (HART, 1982)

$$i(t) = i_0(t)+i_p(t)+i_{DC} \quad (1)$$

$i_0(t)$ is an initial transient associated with the charging of the system's high-frequency capacitance. It should be completed before the first current measurement and will thus be neglected. i_{DC} is the steady-state conduction current. $i_p(t)$ is the polarisation current. The simplest response of a dielectric to a voltage step is the exponential Debye type behaviour for which

$$i_p(t) = Ae^{-bt} \quad (2)$$

where the maximum current A occurs at $t = 0$ and the relaxation time is b^{-1}. Most dielectrics, however, obey the Curie/von Schweidler law

$$i_p(t) = Bt^{-n} \quad (3)$$

where B is the current at $t = 1$ s and n is a positive number. For insulating dielectrics i_{DC} is negligible in comparison with $i_p(t)$, but may be much greater than $i_p(t)$ for lossy dielectrics.

In TDDS a voltage step is applied at time $t = 0$. The current is sampled using an analogue-to-digital convertor (ADC) with an interval T starting at a time t_0 and ending at a time t_f. The capacitance spectrum is given by the Fourier transformation

$$C(f) = C_H+V^{-1}\int_{t_0}^{t_f}\{i(t)-i_{DC}\}\cos(2\pi ft)dt \quad (4)$$

where f is the frequency, C_H is the high-frequency capacitance of the material and V is the applied voltage. The maximum frequency obtainable is the Nyquist frequency $f_N = (2T)^{-1}$; the minimum is t_f^{-1}. In the following Sections several errors introduced by the transformation procedure for the capacitance are discussed. Similar analyses hold for the dielectric loss and conductance.

2 Procedure

2.1 *Discrete steps*

Suppose that the sampled current is i_1 for $t_1 \geq t \geq 0$ and i_2 for $\infty > t \geq t_1$ where i_1 and i_2 are constants. From eqn. 4

$$C(f) = C_H+(i_1-i_2)\sin(2\pi ft_1)/2\pi fV \quad (5)$$

The transformation of this current step varies with frequency as $\sin(2\pi ft_1)/f$. There appears in the capacitance spectrum a wave, the amplitude of which decreases. The digitised current decreases in a series of such steps. The wavy noise in the transformed spectra will be most pronounced at low frequencies. Its relative importance will depend on $\Delta i/i_A$ where Δi is the current step decrement and i_A is the average current sampled. Hence a 12-bit ADC would be superior to an 8-bit ADC in this respect.

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2.2 *Finite time window*

The Fourier transformation of a Debye-type polarisation current is ideally ($0 \leqslant t < \infty$)

$$C(w) = C_H + \frac{Ab}{V(b^2+\omega^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $\omega = 2\pi f$. With a finite window ($t_0 \leqslant t \leqslant t_f$), however, eqn. 4 yields

$$C(\omega) = C_H + \frac{A}{V(b^2+\omega^2)}\{e^{-bt_0}(b\cos\omega t_0 - \omega\sin\omega t_0) - e^{bt_f}(b\cos\omega t_f - \omega\sin\omega t_f)\} \quad . \quad . \quad (7)$$

The additional terms are introduced by the finite window.

The transformation of a Curie/von Schweidler polarisation current is ideally

$$C(\omega) = C_H + \frac{B\pi\omega^{n-1}\sec(n\pi/2)}{2V\Gamma(n)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where Γ is the gamma function. A closed-form solution similar to eqn. 7 cannot be obtained in this case. Instead, Fig. 1 compares the results obtained by numerically integrating $i_p = 100t^{-0\cdot05}\,\mu$A for $0{\cdot}001\,\text{s} \leqslant t \leqslant 3\,\text{s}$ using eqn. 4 with those found by application of eqn. 8. The introduction of wavy noise is clear.

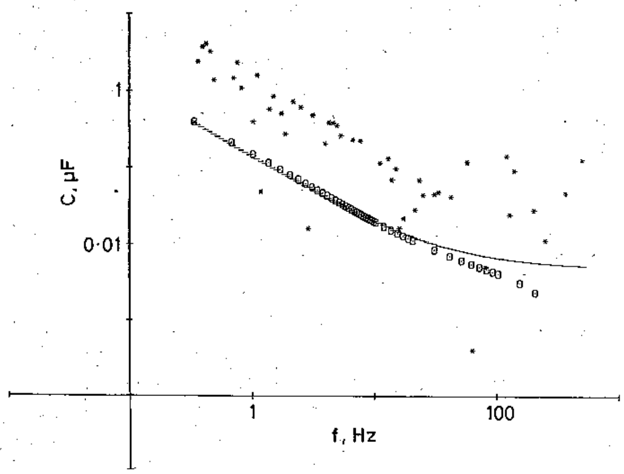

Fig. 1 *Capacitance spectrum for $i_p = 100t^{-0\cdot05}\,\mu A$: (–), ideal spectrum from eqn. 8; (*), spectrum obtained numerically from eqn. 4; (0), spectrum obtained numerically with $\sin\omega t_f = 0$. $C_H = 0{\cdot}005\,\mu F$*

If the transformation is carried out only for frequencies such that $\sin\omega t_f = 0$ and b^{-1} is well within the time window, then eqn. 7 reduces to eqn. 6, with some error at high frequencies. Fig. 1 indicates that the errors are also considerably reduced in the numerical integration of the Curie/von Schweidler response if only frequencies satisfying $\sin\omega t_f = 0$ are used.

2.3 *Discontinuity in the current slope*

Some dielectrics exhibit responses which can be fitted to two sequential Curie/von Schweidler segments (Nakamura *et al.*, 1982; Shimokawa *et al.*, 1981; Hart, 1983*b*). Fig. 2 illustrates the capacitance spectrum corresponding to $i_{p1} = 50t^{-0\cdot02}\,\mu$A for $0{\cdot}001\,\text{s} \leqslant t \leqslant 0{\cdot}1\,\text{s}$ and $i_{p2} = 43{\cdot}5t^{-0\cdot08}\,\mu$A for $0{\cdot}1\,\text{s} \leqslant t \leqslant 3\,\text{s}$. Eqn. 4 was used at frequencies satisfying $\sin\omega t_f = 0$. The spectra obtained from the application of eqn. 8 to these responses separately are presented for comparison. Although good agreement is found with the transformation of i_{p1} at high frequencies and of i_{p2} at low frequencies, a wavy noise has been introduced near $\omega = 0{\cdot}1^{-1}$. The current, but not its slope, is continuous at 0·1 s. The generation of such wavy noise by the transformation of two sequential Debye responses with discontinuous slope can be exhibited in closed form. Although sequential Debye responses are not observed in practice, this fact would indicate that the slope discontinuity is probably also responsible for the noise in the sequential Curie/von Schweidler responses of Fig. 2.

2.4 *Subtraction of the steady-state current*

The transformation for the capacitance uses only the polarisation current $i_p(t) = i(t) - i_{DC}$ in eqn. 4. But for lossy dielectrics it may not be obvious that the polarisation has been completed by t_f. A Curie/von Schweidler polarisation current with n just above zero is easily mistaken for i_{DC}. Let I be the measured current at t_f and $g(t) = i(t) - I$. I and $g(t)$ must serve as the best estimates for i_{DC} and $i_p(t)$. Then

$$\int_{t_0}^{t_f} g(t)\cos\omega t\,dt = \int_{t_0}^{t_f} i(t)\cos\omega t\,dt - \frac{2I}{\omega}\sin\{\omega(t_f - t_0)/2\}\cos\{\omega(t_f + t_0)/2\}$$

The I term vanishes for $\omega = 2m\pi/(t_f - t_0)$, where m is any integer. For such frequencies the transformation is independent of the actual current at t_f and depends only on the measured polarisation current during the interval (t_0, t_f). At other frequencies waves proportional to I appear in the transformation. With lossy, biological dielectrics where $I \gg g(t)$ this noise can be severe. For the selected frequencies it does not matter whether I was really i_{DC} or the tail of a Curie/von Schweidler response. For $t_f \gg t_0 \sin\omega t_f = 0$, and the condition for the reduction of the finite-window noise in Section 2.2 is also satisfied.

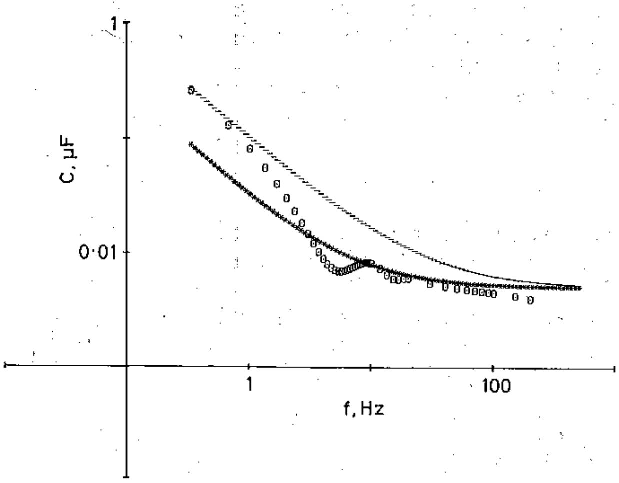

Fig. 2 *Capacitance spectrum with discontinuity in current slope: (*), ideal spectrum for i_{p1} from eqn. 8; (–), ideal spectrum for i_{p2}; (0), spectrum of i_{p1} for $0{\cdot}001\,s \leqslant t \leqslant 0{\cdot}1\,s$ and i_{p2} for $0{\cdot}1\,s \leqslant t \leqslant 3\,s$ obtained numerically from eqn. 4 with $\sin\omega t_f = 0$. $C_H = 0{\cdot}005\,\mu F$*

3 Conclusions

Several precautions should be taken in the Fourier transformation of the polarisation current for lossy dielectrics. The transformations should be made discrete and carried out only for frequencies such that $f = m(t_f - t_0)^{-1}$. If the polarisation response is very weak, so that it occurs in only a few discrete steps with an 8-bit ADC, it may be preferable to use a 12-bit ADC even though the conversion time might be increased. Peaks and waves in the spectra, particularly near the high- and low-frequency ends, should be regarded with suspicion. In general, if there is no structure in the polarisation current there should be none in the transformed spectra. These precautions are not as important for insulating dielectrics for which the polarisation current is dominant.

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